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Contents

1 Teacher’s Edition Assessment: Trigonometry and Right Angles
   1.1 Teacher’s Edition Assessment: Trigonometry and Right Angles 2

2 Teacher’s Edition Assessment: Circular Functions
   2.1 Teacher’s Edition Assessment: Circular Functions 19

3 Teacher’s Edition Assessment: Trigonometric Identities
   3.1 Teacher’s Edition Assessment: Trigonometric Identities 30

4 Teacher’s Edition Assessment: Inverse Functions and Trigonometric Equations
   4.1 Teacher’s Edition Assessment: Inverse Functions and Trigonometric Equations 36

5 Teacher’s Edition Assessment: Triangles and Vectors
   5.1 Teacher’s Edition Assessment: Triangles and Vectors 43

6 Teacher’s Edition Assessment: Polar Equations and Complex Numbers
   6.1 Teacher’s Edition Assessment: Polar Equations and Complex Numbers 55
Chapter 1

Teacher’s Edition Assessment:  
Trigonometry and Right Angles

Introduction

The following document is intended to provide teachers with a variety of assessments for each chapter of the Trigonometry Flex Book. There are pretests for the first five chapters of the text and these are intended, in some cases, to either provide the students with a preview of the upcoming chapter. Other pretests are intended to provide the student with the opportunity to review the skills required to successfully complete the questions in the chapter. In addition to the pretests, there are quizzes provided for each chapter. The quizzes will give the teacher an opportunity to assess the student’s progress as the topics are being presented. The weight associated with these quizzes should be considerably lower than that of a test. If the results achieved on the quizzes are not up to the teacher’s standards, then the areas that reflect little success should be reviewed.

Each chapter also includes a minimum of two tests. The tests include multiple choice questions, computation questions, questions that require the use of technology and application questions. The tests can be administered as written or they can be used in conjunction with tests developed by the teacher.

There is no answer key included in this document. The answer to any mathematical question is only a small percentage of the value of the question. The process employed by the student to arrive at the solution is the most important aspect of question. Every teacher presents his/her method to the students and stresses various points and steps that should be included when answering a question. For this reason, the teacher should develop an answer key and rubric to reflect what the student has been taught and what the teacher expects to see in the solving process.

This course is to be taught over one semester in high school. A semester consists of approximately 19 weeks or 118 hours of instruction. In order to complete the course over this period, time management is critical. The following outline is simply a suggestion that may be helpful.

The assessments presented in this document are designed to assist teachers in preparing assessment materials for the students. Each chapter will include a pretest, one or more quizzes (depending upon the composition of the chapter) and two tests. The pretest may be presented as a preview to enlighten the students about the upcoming topics in the chapter or it may be presented as a review of the necessary skills that students need to successfully complete the exercises in the new chapter. The quizzes are to enable both the teacher and the student to determine whether or not the concepts being taught are being understood. The tests contain a variety of questions that reflect the standards within the chapter and provide the opportunity for the teacher to either use the test as it is written or to pick and choose questions to incorporate into their own tests.
1.1 Teacher’s Edition Assessment: Trigonometry and Right Angles

Pretest
If the following triangles are similar, determine the length of the indicated side.

1. a)

![Diagram of similar triangles with sides labeled 3, x, 4.5, and 5.

b)

![Diagram of similar triangles with sides labeled 3, x, 7, and 5.

c)

![Diagram of a triangle with sides labeled 12.5, b, 4.85, and 6.25.

d)

![Diagram of a slanted triangle with sides labeled 7, 4, c.

Use the Pythagorean Theorem to calculate the length of the indicated side.
Solve the following equations.

3. a) \( \frac{a}{7} = \frac{11}{35} \)
b) \( \frac{3d}{11} = \frac{25}{18} \)
c) \( \frac{51}{9} = \frac{17}{3y} \)
d) \( \frac{1.2m}{5.37} = \frac{21.6}{18.5} \)
e) \( \frac{k}{(1.25)(1.74)} = \frac{(2)(2.64)}{6.75} \)

Simplify the following by first rationalizing the denominator. Express all answers in simplest radical form:
4. a) \( \frac{2\sqrt{5}}{3\sqrt{2}} \)
   b) \( \frac{2}{\sqrt{7} - \sqrt{3}} \)
   c) \( \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{5} + \sqrt{2}} \)

5. For the following triangle, state the length of each side and the measure of each angle.

\[ \angle A = \boxed{\text{________}} \]
\[ \angle B = \boxed{\text{________}} \]
\[ \angle C = \boxed{\text{________}} \]
\[ \overline{AB} = \boxed{\text{________}} \]
\[ \overline{AC} = \boxed{\text{________}} \]
\[ \overline{BC} = \boxed{\text{________}} \]

This is the first quiz for this chapter. It consists of questions that reflect the topics of the first three lessons.

**Basic Functions**

**Angles in Triangles**

**Measuring Rotation**

**Quiz One**

**Part A – Multiple Choice** – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _______ What is the domain and range for the following relation?
2. In the following figure, what is the value of ‘x’?

![Diagram of a triangle with sides 7 in, 4 in, and 5 in, and angles labeled as 45°, 72°, and x°.]

a) 5.6 in.

b) 7.0 in.

c) 8.75 in.

d) 2.0 in.

3. What is the value of 286.88° expressed in degrees, minutes and seconds?

a) 286°9′

b) 286°8′8″

c) 286°48′48″

d) 286°52′48″

4. In the following triangle what is the measure of \( \angle A \)?

![Diagram of a triangle with angles labeled as 45°, 72°, and x°.]

a) 45°
b) $63^\circ$

c) $72^\circ$

d) $180^\circ$

5. Which of the following relations is not a function?

a) $(1, 4), (1, 7), (1, 10), (1, 13), (1, 16)$

b) $(1, 4), (2, 4), (3, 4), (4, 4), (5, 4)$

c) $(1, 2), (-1, 2), (2, 1), (-2, -1)$

d) $(0, 0), (1, 1), (-1, -1), (2, 4), (-2, 4)$

6. Which of the following best represents all the angles that are coterminal with $45^\circ$?

a) $x = 45^\circ$

b) $x = 180^\circ - 45^\circ$

c) $x = 45^\circ + 360^\circ$

d) $x = 45^\circ + 360^\circ k, k \in I$

Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.

1. A local arena has a seating capacity for 4,000 people. The attendance at the first hockey game at the beginning of each month has been recorded.

<table>
<thead>
<tr>
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<td>Attendance</td>
<td>2,500</td>
<td>2,750</td>
<td>2,900</td>
<td>3,100</td>
<td>3,450</td>
<td>3,500</td>
<td>3,875</td>
<td>3,700</td>
<td>3,000</td>
<td>2,200</td>
</tr>
</tbody>
</table>

a) Use technology to produce a scatter plot of the given data. Sketch the scatter plot in the space below.

b) What shape is represented by the scatter plot?

c) Determine an equation to model this problem.

2. Do the following graphs represent functions? Justify your answer.

a)
3. Convert $47^\circ28'40''$ to degrees. Show all the necessary work.

This is the second quiz for this chapter. It consists of questions that reflect the topics of lessons four, five and six.

**Defining Trigonometric Functions**

**Trigonometric Functions of Any Angle**

**Relating Trigonometric Functions**

Note: A major test given at the end of the chapter should include Lesson 7. The reason it is not included in the quiz is to allow some time between the quiz and the major test.

**Quiz Two**

**Part A – Multiple Choice** – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. If $\theta$ is an angle in standard position and the point $(-3,4)$ is a point on the terminal side, what is the value of the secant function?
   a) $\sec \theta = -\frac{3}{4}$
   b) $\sec \theta = -\frac{4}{3}$
   c) $\sec \theta = -\frac{5}{3}$
   d) $\sec \theta = -\frac{3}{5}$

2. Which of the following would be used to calculate $\overline{BC}$?

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3. What is the length of the unknown side in the following triangle?

4. If \(\sin A = .8480\), then what does \(\cos A\) equal?

5. In what two quadrants is the cosecant function negative?

6. Which of the following angles is coterminal with an angle of \(55^\circ\)?
b) 235°
c) 305°
d) 415°
7. What is the measure of side “x” in the following diagram?

![Triangle Diagram]

a) x = 9.0
b) x = 27
c) x = 5.8
d) x = 4.1
8. What is the measure of the reference angle for an angle of 120° drawn standard position?
   a) 480°
   b) 60°
   c) 30°
   d) 90°
9. Rearrange the following equation by solving for “A”: \(12 = 9 - 6 \cos A\)
   a) \(\cos^{-1}(4) = A\)
   b) \(\frac{3}{-5 \cos} = A\)
   c) \(12 - 3 \cos = A\)
   d) \(\cos^{-1}(-.5) = A\)
10. What is the exact value of \(\cos 315° + \tan 225°\)?(Use the special triangles).
   a) \(\frac{\sqrt{2}}{2}\)
   b) \(\frac{1}{\sqrt{2}} + 2\)
   c) \(\frac{2 + \sqrt{2}}{2}\)
   d) \(\frac{\sqrt{2} - 1}{2}\)

Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.

1. Determine the other five trigonometric ratios, given \(\cos \theta = -\frac{5}{15}\).
2. If \(\cos \theta < 0\) and \(\cot \theta = -\frac{4}{3}\), determine the value of \(\sec \theta\) and \(\csc \theta\).
3. What is the height of a building that casts a shadow 28.4 m long when the angle of elevation of the sun is 51.6°?

4. While looking out the window of our classroom, 4.6 m above the ground, you wave to your friend whom you see at an angle of depression of 10°. Then you see another friend at an angle of depression of 3° coming behind your first friend in the same path. Assuming that the ground is level, how far apart are your two friends?

5. What is the exact value of \( \sin 240° - \cos 240° \)? (Use the special triangles).

6. Solve the following triangle:

   ![Diagram](image)

This is a test for the entire chapter Trigonometry and Right Angles.

**Test One**

**Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.**

1. ______ The following diagram shows a ladder leaning against a house. Which of the following would you use to find the height the ladder reaches up the wall?
   
   ![Diagram](image)

   a) Pythagorean Theorem  
   b) Sine  
   c) Cosine  
   d) Tangent

2. ______ The lot of land shown at the right borders on Hornes Road in Mira. What is the length of
the property along the road?

q) 27 m
b) 69.9 m
c) 127.6 m
d) 4892.3 m

3. Which of the following best represents the domain and range of graph shown?

a) domain = \{x \mid -1 \leq x \leq 5, x \in I\}
   range = \{y \mid -15 \leq y \leq 3, y \in R\}

b) domain = \{x \mid x \in R\}
   range = \{y \mid y \leq 3, y \in R\}

c) domain = \{x \mid -1 \geq x \geq 5, x \in I\}
   range = \{y \mid -2 \geq y \geq 3, y \in R\}

d) domain = \{x \mid x \in R\}
   range = \{y \mid y \in R\}

4. What name is given to a triangle that has two sides equal in length and two base angles equal?
a) equilateral triangle
b) scalene triangle

c) oblique triangle

d) isosceles triangle

5. Jim’s One Stop is advertising a special for student cell phones. The special will consist of a charge of $15 per month and 4 cents per minute. Which of the following equations best describe the plan where \( x \) is the time in minutes and \( c \) is the monthly cost?

a) \( c = 15x + 4 \)

b) \( c = 4x + 15 \)

c) \( c = .04x + 15 \)

d) \( c = 15x + .04 \)

6. For the above cell phone service, what would be the cost of 250 minutes?

a) 

b) 

c) 

d) 

7. If the following triangles are similar, what is the length of the indicated side?

a) 12.6

b) 7.0

c) 5.0

d) \( \sqrt{74} \)

7. If \( \angle A = 38^\circ \), what is the value of \( \sec A \)?

a) 0.7880

b) 1.6243

c) 1.2690

d) 0.6157

8. What is the measure of 37.425° expressed in degrees, minutes and seconds?

a) 37°42.5′

b) 37°42′5′

c) 37°25.5′

d) 37°25′30″

9. Which angle is not coterminal with an angle of 150°?

a) \(-210^\circ\)
b) $510^\circ$

c) $-450^\circ$

d) $870^\circ$

10. If $\theta$ is an angle in standard position and the point $(-6, 8)$ is a point on the terminal side, what is the value of the cosecant function?

a) $\csc \theta = -0.8000$

b) $\csc \theta = 1.2500$

c) $\csc \theta = 1.6667$

d) $\csc \theta = 0.6000$

**Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.**

1. The Silver Dollar Restaurant has banquet facilities for a maximum of 200 people. When the banquet manager quotes a price for a banquet, the price of the room as well as the cost of the meal is included. A banquet for 120 people will cost $1300 and a banquet for 80 people will cost $900.

a) Using technology, plot a graph of cost versus the number of people. Sketch the graph in the space provided.

b) Use the graph to determine the cost of a banquet for 150 people.

c) Determine an equation to model this function.

d) Use the equation to calculate the cost of a banquet for 95 people.

2. a) Express the measure of $48^\circ 30' 36''$ to decimal form in degrees.

b) Express the measure $36.70$ to degrees, minutes seconds.

3. In each of the following, calculate the length of the indicated side to the nearest tenth.

a) ![Diagram](image1)

b) ![Diagram](image2)
4. If $\theta$ is an angle in standard position and the point $(8, -6)$ is a point on the terminal side, determine the value of the six trigonometric functions of $\theta$.

5. A spider crawling down a wall spots supper on the ground at an angle of 160 with the wall. After crawling 16 inches further down the wall, supper still hasn’t moved but now the angle with the wall is 280. How far is supper from the wall?

6. Sketch each of the following angles in standard position and determine the measure of the reference angle.
   a) $315^\circ$
   b) $-120^\circ$
   c) $150^\circ$

**Test Two**

**Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.**

1. _____ Which of the following equations could be used to determine the measure of $\angle A$  
   a) $A^2 = 6.7^2 + 9.4^2$
   b) $\tan \angle = \frac{9.4}{6.7}$
   c) $\sin \angle \frac{9.4}{6.7} = \frac{\sin 90^\circ}{6.7}$
   d) $\cos \angle = \frac{6.7}{9.4}$

2. _____ If $\tan \theta = 1.8807$, what is the value of $\sin \theta$?
3. "A Cut above the Rest" is a company that does lawn grooming. They offer customers three plans from which to choose. Plan A charges a down payment of $200 and $2 per hour. What would be the cost of 12 hours of lawn maintenance?
   a) $200.00
   b) $24.00
   c) $2402.00
   d) $224.00

4. What is the measure $122^\circ44'15''$ expressed in degrees in decimal form?
   a) 122.7375°
   b) 122.4415°
   c) 122.4475°
   d) 122.7315°

5. Why is the following graph a function?
   a) My vertical line cuts the graph only once.
   b) Any line would cut the curve only once.
   c) There is only one $x$-value for each $y$-value.
   d) Any vertical line would cut the curve at exactly one point.

6. Which of the following set of points do NOT represent a function?
   a) $(-2, 5); (-1, 5); (0, 5); (1, 5); (2, 5)$
   b) $(5, -2); (5, -1); (5, 0); (5, 1); (5, 2)$
   c) $(-2, 4); (-1, 2); (2, -4); (1, -2)$
   d) $(-2, 3); (-1, 5); (0, 7); (1, 9); (2, 11)$

7. Which of the following pair of angles is coterminal with an angle of $210^\circ$?
   a) $30^\circ$ and $150^\circ$
   b) $30^\circ$ and $-150^\circ$
   c) $-150^\circ$ and $570^\circ$
   d) $30^\circ$ and $930^\circ$
8. If the following pair of triangles is similar, what is the measure of the indicated side?

\[ x = 54 \]
\[ x = 18 \]
\[ x = 24 \]
\[ x = 1.5 \]

9. In the following triangle, what is the measure of \( \angle CBD \)?

\[ \angle CBD = 51^\circ \]
\[ \angle CBD = 129^\circ \]
\[ \angle CBD = 39^\circ \]
\[ \angle CBD = 36^\circ \]

10. In what quadrants is the cotangent function positive?

a) 1st and 2nd
b) 1st and 4th
c) 2nd and 4th
d) 1st and 3rd

Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.

1. If \( \theta \) is an angle in standard position and the point \((-3, -6)\) is a point on the terminal side, determine the value of the six trigonometric functions of theta. Express all answers in simplest radical form.

2. In each of the following, calculate the length of the indicated side to the nearest tenth.
3. While exploring the woods at the end of a country road, two hunters spotted a fire in the distance. From where they were standing, they estimated an angle of elevation of $15^\circ$ to the top of the tower. Moving 10 m closer to the tower, they now estimated the angle of elevation to be $18^\circ$. How high is the tower?
4. A car rental agency charges $28.00 for the first 20 miles and $0.40 for each additional mile.
   a) Using technology, plot a graph of cost versus distance. Sketch the graph in the space provided.
   b) Use the graph to determine the cost of driving 100 miles.
   c) Determine an equation to model this function.
   d) Use the equation to determine the distance driven, if the bill was $112.00.

5. a) For each of the following angles, determine the measure of two coterminal angles.
   a) 65°
   b) 145°
   c) 315°
   b) For each of the angles given above, write a general rule for determining the measure of all the coterminal angles.

6. From a point 63 yards from the foot of a building, the angle of elevation to the top of the building is 38°. From the same point, the angle of elevation to the top of a flagpole on top of the building is 42°. What is the length of the flagpole?
Chapter 2

Teacher’s Edition Assessment: Circular Functions

2.1 Teacher’s Edition Assessment: Circular Functions

Pretest

An angle can be measured in degrees, minutes and seconds. The measure of an angle can also be stated in terms of the length of the arc that subtends the angle at the center of a circle. An angle subtended at the center of a circle by an arc equal in length to the radius of the circle has a measure of one radian.

\[ \theta = \frac{\text{arc length}}{\text{radius}} = \frac{a}{r}. \]

In order to convert from degree measure to radian measure, the relationship between degrees and radians must be used. One revolution is equal to \(360^\circ\) and in radian measure one revolution equals \(2\pi\) radians. If \(2\pi \text{rad} = 360^\circ\) then \(\frac{2\pi \text{rad}}{2} = \frac{360^\circ}{2} \implies \pi \text{rad} = 180^\circ\). Using this information, answer the following questions:

1. Calculate the degree measure of the angles whose radian measures are:
   a) \(\frac{\pi}{3}\)
   b) \(\frac{\pi}{2}\)
   c) \(\frac{5\pi}{4}\)
   d) \(-3\pi\)
   e) \(\frac{4\pi}{3}\)

2. Calculate the radian measure of the angles whose degree measures are:
   a) \(30^\circ\)
   b) \(1^\circ\)
   c) \(315^\circ\)
   d) \(225^\circ\)
   e) \(-115^\circ24'\)

3. A small electrical motor rotates at 2200 r/min.
   a) Express this angular velocity in radians per second.
b) Calculate the distance a point 0.9 inches from the center of rotation travels in 0.009 seconds.

4. An automobile travels at 100 miles per hour.
   a) Calculate the angular velocity of a tire with a radius of 15 inches.
   b) Through what angle will the tire turn in 30 seconds at this speed?

5. For each of the following graphs, the pre-image graph is Graph 1. In words, explain and describe what transformation was applied to this graph to produce Graph 2.

a)  

b)  

---
c)

Graph 1

Graph 2

30 60 90 120 150 180 210 240 270 300 330 360

-4 -3 -2 -1 0 1 2 3 4


d)

-2\pi -\frac{5\pi}{3} -\frac{4\pi}{3} -\pi \frac{2\pi}{3} \frac{\pi}{3} \pi \frac{4\pi}{3} \frac{5\pi}{3} 2\pi

-5 -4 -3 -2 -1 0 1 2 3 4

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Radian Measure

Applications of Radian Measure

Circular Functions of Real Numbers

Linear and Angular Velocity

The quizzes for this chapter are both in the format of multiple choice questions. This type of quiz is acceptable as long as all of the outcomes are included in the questions. The students will have to have a working knowledge of the concepts in order to be successful but will not be required to show the necessary computations.

Quiz One

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _____ What is the value of $73.68^\circ$ expressed in radian measure?
   a) $73.68^\circ \approx 23.4531\text{rad}$
   b) $73.68^\circ \approx 1.2860\text{rad}$
   c) $73.68^\circ \approx 23.4725\text{rad}$
   d) $73.68^\circ \approx 0.7776\text{rad}$

2. _____ What is the exact value of $(\tan 300^\circ)(\sec 300^\circ)$?
   a) $2\sqrt{3}$
   b) $\frac{2\sqrt{3}}{3}$
   c) $\frac{-\sqrt{3}}{2}$
   d) $-2\sqrt{3}$

3. _____ If a compass has 16 bearings and the bearing of WNW is bearing number 13, what angle, in degrees, describes this bearing?
   a) $22.5^\circ$
   b) $292.5^\circ$
   c) $27.69^\circ$
   d) $315^\circ$

4. _____ What is the measure $\frac{13\pi}{20}$ in degrees?
   a) $117^\circ$
   b) $2.04^\circ$
   c) $37.2^\circ$
   d) $243^\circ$

5. _____ What is measure of the radius of a circle that has a central angle of $298^\circ$ and an arc length of 50 inches?
a) 260 in
b) 5.96 in
c) 9.62 in
d) 57.3 in

6. _____ What is the area of the sector of a circle with a central angle of 218° and a radius of 5.65 inches?
   a) $A \approx 10.75 \text{ in}^2$
   b) $A \approx 121.46 \text{ in}^2$
   c) $A \approx 5.38 \text{ in}^2$
   d) $A \approx 60.73 \text{ in}^2$

7. _____ What is the exact value of \((\cos \frac{5\pi}{6})(\sin \frac{2\pi}{3}) - (\sin \frac{5\pi}{6})(\cos \frac{2\pi}{3})\)?
   a) $-\frac{1}{2}$
   b) $-\frac{2\sqrt{3}}{3}$
   c) 0
   d) -2

8. _____ If a person on a hang glider is moving in a horizontal, circular arc of radius 95.0 meters with an angular velocity of 0.125 rad/s, what is the person’s linear velocity?
   a) $v = 760 \text{ m/s}$
   b) $v = 11.3 \text{ m/s}$
   c) $v = 13.26 \text{ m/s}$
   d) $v = 4.83 \text{ m/s}$

9. _____ What is the average angular velocity in a U-turn made by a vehicle in 6.0 seconds?
   a) $W \approx 9.55 \text{ rad/s}$
   b) $W \approx 30 \text{ rad/s}$
   c) $W \approx 0.52 \text{ rad/s}$
   d) $W \approx 1.63 \text{ rad/s}$

10. _____ What is the measure in degrees of the central angle of a circle that has an arc length of 3.19 feet and a radius of 2.29 feet?
    a) $\theta \approx 1.39^\circ$
    b) $\theta \approx 79.81^\circ$
    c) $\theta \approx 4.36^\circ$
    d) $\theta \approx 88.27^\circ$
Graphing Sine and Cosine Functions

Translating Sine and Cosine Functions

General Sinusoidal Functions

Quiz Two

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _____ What is the phase shift for the following sinusoidal function?
   \[ y = -3 \sin(5(x + 30^\circ)) + 7 \]
   
   a) \(-3\)
   b) \(\frac{1}{5}\)
   c) \(-30^\circ\)
   d) 7

2. _____ Which of the following graphs represents the equation? \(-y = \cos x\)
   
   a) 
   b) 
   c)
3. _____ Given the following equation where \( x \) represents time in seconds and \( y \) represents height in meters, what is the height at 42 seconds?

\[ y = 6 - 2 \cos 3(x - 5) \]

- a) 3.5m
- b) 6.7m
- c) 1.7m
- d) 12.3m

4. _____ If the graph of \( y = \cos x \) undergoes a vertical stretch of \( \frac{1}{4} \), a vertical translation of \(-6\), a horizontal stretch of 2 and a horizontal translation of \(15^\circ\), which of the following equations would best model the graph?

- a) \( y = \frac{1}{4} \cos \frac{1}{2}(x - 15^\circ) - 6 \)
- b) \( y = \frac{1}{4} \sin \frac{1}{2}(x - 15^\circ) - 6 \)
- c) \( y = 4 \cos -2(x - 15^\circ) - 6 \)
- d) \( y = 4 \cos 2(x - 15^\circ) + 6 \)

5. _____ What is the name given to the part of a sinusoidal curve that is located midway between the highest and the lowest point?

- a) the phase shift
- b) the sinusoidal axis
- c) the diameter
- d) the radius

6. _____ What is the horizontal stretch of a sinusoidal curve that has a period of \(120^\circ\)?

- a) 3
- b) 2
- c) \( \frac{1}{2} \)
- d) \( \frac{1}{3} \)

7. _____ If the equation of a sinusoidal function expressed in standard form is \( y = -6 \sin 7(x - 20^\circ) - 5 \), what is the equation of the sinusoidal axis?

- a) \( y = -6 \)
- b) \( y = -5 \)
- c) \( y = 5 \)
- d) \( y = 20 \)
8. _____ What is the vertical translation of \( y = \sin x \) in the following graph?

![Graph of \( y = \sin x \)](image)

a) V.T. = 3  
b) V.T. = 7  
c) V.T. = None  
d) V.T. = 5

In the following diagram, an old car tire is swinging back and forth on a rope. The following graph compares its distance from the vertical wall and the time. Using the diagram answer the following two (2) questions:

![Diagram of tire swinging](image)

9. ______ How far from the wall is the tire when it is not moving (just hanging there)?
   a) 0.6m  
b) 1.7m  
c) 2.3m  
d) 2.8m

10. ______ How long does it take for the tire to swing from its closest distance to the wall to its farthest distance from the wall?
    a) 1.2 seconds  
b) 2.4 seconds  
c) 2.8 seconds  
d) 1.4 seconds
1. Study the following graph, list the transformations of $y = \cos x$ and write its equation in standard form.

![Graph of $y = \cos x$]

2. The following diagram represents the floor area of a hallway such that the outside and the inside are circular arcs. What is the floor area of this hallway?

![Diagram of hallway]

3. For the sinusoidal curve that has the equation $y = -5 \sin 3(x - 5^\circ) - 2$, list the transformations of $y = \sin x$ and use those transformations to sketch a graph of the sinusoidal function.

![Graph of sinusoidal function]

4. A highway exit is a circular arc 990 feet long with a central angle of 82.3°. What is the radius of the curvature of the exit?

5. For each of the following, evaluate the given expression and express the exact answer in simplest radical form.
   a) $\cos^2 150^\circ + \tan^2 60^\circ + \sin^2 30^\circ$
   b) $(\sec \frac{\pi}{3})(\sin \frac{\pi}{3}) - (\csc \frac{\pi}{3})(\tan \frac{\pi}{4})$

6. Study the following graph of a sinusoidal curve.
a) For the above graph, identify the four parts of a cosine curve.

b) Write the transformations of \( y = \cos x \). Show all necessary calculations!

c) Write the equation as a cosine function.

7. An airplane propeller blade is 4.2 feet long and rotates at 2200 r/min. What is the linear velocity of a point on the tip of the propeller blade?

8. The all terrain vehicles that travel on the ice roads in Alaska, have tires that are 3.66 meters in diameter. If the vehicle travels at a speed of 5.6 km/h, what is the angular velocity of the vehicle in revolutions per minute?

**Test Two**

Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. The arm of a windshield wiper and its blade are in a straight line. The arm of the windshield wiper is 12.8 inches long and is attached to the center of a 15 inch blade. What area of the windshield is cleaned by the wiper if it passes through 110.0° arcs?

2. Study the following sinusoidal graph.

   a) List the parts of the sinusoidal curve for a sine function.

   b) List the transformations of \( y = \sin x \).

   c) Write the equation in terms of the sine function to model this graph.

3. For each of the following, evaluate the given expression and express the exact answer in simplest radical form.

   a) \( \cos^2 \frac{3\pi}{4} - \sin^2 \frac{\pi}{3} \)

   b) \( \left( \cos 315° \right) \left( \sin 210° \right) - \left( \cos 150° \right) \left( \cos 135° \right) \)
4. A helicopter blade is 8.25 feet long and is rotating at 420 r/min. What is the linear velocity at the tip of the blade?

5. Study the following sinusoidal function: \( y = 5 + 3 \cos 9\left(x - \frac{x}{9}\right) \)
   a) List the transformations of \( y = \cos x \)
   b) Use these transformations of \( y = \cos x \) to sketch one cycle of the curve.

6. At the Fortress of Louisbourg, a waterwheel is used to generate power. The paddles of the waterwheel are 11.25 feet long. The speed at the end of a paddle is one-fourth that of the water. If the water is flowing at a rate of 6.5 ft/s, what is the angular velocity of the waterwheel?

7. A shaded sector of a pie chart has a radius of 2.44 inches. If the perimeter of the shaded sector is 7.32 inches, what is the measure, in degrees, of the central angle of the sector?

8. A remote control helicopter was being tested for its consistent flying ability. Under correct monitoring; the helicopter could fly up and down in a sinusoidal pattern. To demonstrate this movement, a graph was drawn to show the helicopter’s height at various times. The graph showed that the helicopter reached its maximum height of 20 meters in 3 seconds and at 11 seconds it was at its minimum height of 2 meters. Draw a graph to represent the flight pattern of the helicopter and write an equation for the graph.
Chapter 3

Teacher’s Edition Assessment: Trigonometric Identities

3.1 Teacher’s Edition Assessment: Trigonometric Identities

For this chapter, the pretest will involve questions that represent the algebraic skills required to process the topics throughout the chapter. This chapter involves extensive computations and manipulations with formulas.

Pretest

1. Solve the following equations:
   a) $3(x - 2) + 5 = x$
   b) $\frac{y-1}{2} + \frac{y-3}{4} = \frac{y-5}{6}$
   c) $m^2 - m - 20 = 0$
   d) $5x^2 + 1 = 6x$
   e) $a^2 + 3a = 5$
   f) $\frac{1}{n} + \frac{2}{n+3} = \frac{9}{10}$

2. Factor the following:
   a) $2x^2 - 7x - 15$
   b) $2ab + 4ac - 6ad$
   c) $10m^2 - 19m + 6$
   d) $2x^2 - 5x^2 - 42x$
   e) $4x^2 - 25y^2$
   f) $18c^2 - 51c + 8$

3. Expand and simplify:
   a) $(y + 3)(2y^2 - 7y + 4)$
   b) $2(m - 1)(m + 5) - 4(m - 5)^2$
4. Rationalize the denominator of each of the following and express all answers in simplest radical form.

a) \(\frac{3}{2\sqrt{2}}\)

b) \(\frac{3\sqrt{2}-1}{\sqrt{3}}\)

c) \(\frac{1}{\sqrt{2}+\sqrt{3}}\)

d) \(1 + \frac{3\sqrt{2}}{2\sqrt{3}}\)

5. Solve each of the following for the indicated variable:

a) \(A = \frac{1}{2} \cdot bh \rightarrow h\)

b) \(\frac{a}{n} + \frac{c}{p} = 1 \rightarrow b\)

c) \(d = \frac{3}{2}(y+1) \rightarrow y\)

Fundamental Identities

Verifying Identities

Sum and Difference Identities for Cosine

Sum and Difference Identities for Sine and Tangent

Note: For the quiz, you may want to provide the students with the formulas for the various identities that are presented in the above lessons. The quiz would then test their working knowledge of the formulas. For the tests, the students would have to demonstrate the ability to recall the correct formula for the question as well as their ability to perform the proper computations. This is merely a suggestion with the final format being the decision of the teacher.

Quiz One

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _____If an angle is in standard position and the terminal side of the angle passes through \((-3, 8)\), of what quadrant angle are the signs of the trigonometric functions?

a) first - Quadrant angle

b) second - Quadrant angle

c) third - Quadrant angle

d) fourth - Quadrant angle

2. _____In which quadrant does the terminal side of \(\theta\) lie if \(\cos \theta > 0\) and \(\tan \theta < 0\)?

a) first Quadrant

b) second Quadrant

c) third Quadrant

d) fourth Quadrant

3. _____What is the value of \(\tan \theta\) when \(\sin \theta = -0.5736\) and \(\cos \theta > 0\)?
4. ______ Which one of the following is NOT a trig identity?
   a) \( \tan x = \frac{\sin x}{\cos x} \)
   b) \( \csc x = \frac{1}{\sin x} \)
   c) \( 1 = \sin x + \cos x \)
   d) \( \sec^2 x = \tan^2 x + 1 \)

5. ______ What two expressions represent \( \tan^2 x \)?
   a) \( (\sec^2 x) \) and \( \frac{1}{\sin^2 x} \)
   b) \( (\sec^2 x + 1) \) and \( \frac{\cos^2 x}{\sin^2 x} \)
   c) \( \left(\frac{1}{\cot x}\right) \) and \( \sec^2 x - 1 \)
   d) \( \frac{\sin^2 x}{\cos^2 x} \) and \( \left(\frac{1}{\cot^2 x}\right) \)

6. ______ What is the exact value of \( \cos(45^\circ - 30^\circ) \)?
   a) \( \sqrt{6} + \sqrt{2} \)
   b) \( \sqrt{6} - \sqrt{2} \)
   c) \( \sqrt{3} + \sqrt{2} \)
   d) \( \sqrt{3} - \sqrt{2} \)

7. ______ What single term can be used to represent \( \cos 23^\circ \cos 67^\circ - \sin 23^\circ \cos 67^\circ \)?
   a) \( \sin 44^\circ \)
   b) \( \cos 44^\circ \)
   c) \( \cos 90^\circ \)
   d) \( \sin 90^\circ \)

8. ______ What single term can be used to represent \( \frac{\tan(x-y) + \tan y}{1 - \tan(x-y)\tan y} \)?
   a) \( -\tan x \)
   b) \( \tan x \)
   c) \( -\tan y \)
   d) \( \tan y \)

Part B - Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. If \( \cos \theta = -\frac{1}{2} \) and \( \sin \theta > 0 \), determine the value of \( \tan \theta \)
2. Prove the identity \( \frac{\csc x \cos x}{\cot^2 x} = \tan x \).
3. Use the sum formula for the sine function to determine the value of \( 105^\circ \). (Hint: use 2 special angles)
4. Express \( \cos(2x - y)\cos y - \sin(2x - y)\sin y \) as a single term.
Double-Angle Identities

Half-Angle Identities

Product-and-Sum, Sum-and-Product, and Linear Combinations of Identities

Quiz Two

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. ____ If \( \cos \beta = \frac{3}{5} \) for an angle in the 4th quadrant, what is the value of \( \beta \)?
   a) \(-\frac{3}{5}\)
   b) \(\frac{3}{5}\)
   c) \(\frac{4}{5}\)
   d) \(-\frac{4}{5}\)

2. ____ If \( \beta = \frac{\pi}{6} \) and the double angle identity for tangent were being applied to \( \beta \), what would be the value of the angle being evaluated?
   a) \(\tan \frac{\pi}{6}\)
   b) \(\tan \frac{\pi}{3}\)
   c) \(\tan \frac{\pi}{12}\)
   d) \(\tan \pi\)

3. ____ What single term is equal to \( \cos^2 2a - \sin^2 2a \)?
   a) \(\sin^2 2a\)
   b) \(\cos^2 2a\)
   c) \(\cos 4a\)
   d) \(\sin 4a\)

4. ____ What formula can be used to calculate the exact value of 165°?
   a) \(\sqrt{\frac{1+\cos 330°}{2}}\)
   b) \(\sqrt{\frac{1-\cos 330°}{2}}\)
   c) \(\sqrt{\frac{1-\cos 330°}{2}}\)
   d) \(\sqrt{\frac{1+\cos 330°}{2}}\)

5. ____ If \( \sin \theta = \frac{2}{3} \) and the terminal side of the angle is in the 1st quadrant, what is the exact value of \( 2\theta \)?
   a) \(\frac{\sqrt{5}}{3}\)
   b) \(\frac{2\sqrt{3}}{3}\)
   c) \(\frac{2}{3}\)
d) 4\sqrt{5}/9

Part B - Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

6. Find the value of \( \tan 2x \) if \( \sin x = \frac{1}{2} \) and the terminal side of the angle is in the 2nd quadrant.

7. Verify the following identity: \( 2 + \frac{\cos 2\theta}{\sin^2 \theta} = \csc^2 \theta \)

8. Use a calculator to verify the value found by using the double-angle formula to determine \( \sin 258^\circ \) by using functions of 129\(^\circ\).

9. Use the half-angle formula to simplify the expression \( \sqrt{\frac{1-\cos 6x}{2}} \).

10. Express the product \( (\sin 3\theta)(\sin 2\theta) \) as a sum.

Test One

Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. Prove the following identities by working with the left side of the equation.
   a) \( \frac{1-\sin \theta}{\sin \theta \cot \theta} = \frac{\cos \theta}{1+\sin \theta} \)
   b) \( \sec^2 x \csc^2 x = \sec^2 x + \csc^2 x \)

2. Determine the exact value of \( \cos 15^\circ \) by using \( 15^\circ = 60^\circ - 45^\circ \).

3. Evaluate the given function \( \sin(\alpha + \beta) \) by using the following information: \( \sin \alpha = \frac{4}{5} \) (1st quadrant) and \( \cos \beta = -\frac{12}{13} \) (2nd quadrant)

4. Determine the value of \( \tan 120^\circ \) by using the function of 60\(^\circ\).

5. For this diagram, show that \( x = 2l \sin^2 \frac{1}{2} \theta \).

![Diagram of a triangle with labels x, l, and \( \theta \)]

6. Prove the identity \( (\sin x + \cos x)^2 = 1 + \sin 2x \) and verify the identity by using technology to compare the graph of the left side with the graph of the right side. Sketch the results below.

Test Two

Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. Rewrite the expression \( \frac{\csc x}{\tan x + \cot x} \) as a single term.
2. Use half-angle identities to determine the exact value of \( \cos\left(\frac{\pi}{8}\right) \).

3. Solve the following equations algebraically to determine exact solutions over the interval \([0, 2\pi)\)
   
a) \( \sin 2x + \sin 4x = 0 \)
   
b) \( \cos 2x + \sin x = 0 \)

4. Rewrite the following expression as the tangent of an angle.
   
   \[
   \frac{\tan 23^\circ + \tan 55^\circ}{1 - \tan 23^\circ \tan 55^\circ}
   \]

5. Find the exact value of \( \tan(x - y) \) if \( \sin x = \frac{8}{17} \) and \( \cos y = \frac{2}{5} \) such that \( 0 < x < \frac{\pi}{2} \) and \( 0 < y < \frac{\pi}{2} \).

6. An alternating current \( I \) is given by the equation \( i = i_0 \sin(Wt + \alpha) \). Show that this equation can also be expressed as:
   
   \( i = i_1 \sin Wt + i_2 \cos Wt \) where \( i_1 = i_0 \cos \alpha \) and \( i_2 = i_0 \sin \alpha \)
Chapter 4

Teacher’s Edition Assessment: Inverse Functions and Trigonometric Equations

4.1 Teacher’s Edition Assessment: Inverse Functions and Trigonometric Equations

Pretest

1. State the values of a, b and c for the following quadratic functions:
   a) \( y = 2x^2 + 3x - 7 \)
   b) \( y = 5x - 3 + 7x^2 \)
   c) \( y + 3 = x^2 + 2x \)

2. Solve the following equations:
   a) \( 3(m - 4) - 6 = 5x - 12 \)
   b) \( (x - 1)(x + 3) = (x + 2)(x + 1) \)
   c) \( 4 - (y - 3) + 2(3y - 5) = 6 \)
   d) \( 2w^2 + 11w - 21 = 0 \)
   e) \( 7n^2 = 2n + 2 \)

3. Factor the following (if possible):
   a) \( 4x^2 - 9 \)
   b) \( 30c^2 + c - 20 \)
   c) \( 12g^2 - 25g + 12 \)
   d) \( 10 - 11x + 3x^2 \)
   e) \( 4x^2 + 20x + 25 \)

4. If a function is invertible, its graph and the graph of its inverse will be symmetrical about the line \( y = x \). Using technology, determine which of the following are invertible. Hint: Graph the given function, graph
the function \( y = x \) and then use the DRAW mode and scroll down to Draw Inverse. Enter the function and the calculator will draw the inverse if one exists.

a) \( y = \sqrt{x + 1} \)

b) \( y = (x + 2)^2 + 3 \)

c) \( y = (x + 2)^2, x \geq -2 \)

5. Which of the following best represents the domain and range of graph shown?

A. \[ \text{domain} = \{x | -1 \leq x \leq 5, x \in \mathbb{I}\} \]
   \[ \text{range} = \{y | -15 \leq y \leq 3, y \in \mathbb{R}\} \]

B. \[ \text{domain} = \{x | x \in \mathbb{R}\} \]
   \[ \text{range} = \{y | y \leq 3, y \in \mathbb{R}\} \]

C. \[ \text{domain} = \{x | -1 \geq x \geq 5, x \in \mathbb{I}\} \]
   \[ \text{range} = \{y | -2 \geq y \geq 3, y \in \mathbb{R}\} \]

D. \[ \text{domain} = \{x | x \in \mathbb{R}\} \]
   \[ \text{range} = \{y | y \in \mathbb{R}\} \]

General Definitions of Inverse Trigonometric Functions

Ranges of Inverse Circular Functions

Derive Properties of the other Five Inverse Circular Functions in terms of Arctan

Quiz One

1. State the domain and range of the functions represented by the following mapping diagrams. Which diagrams show A mapping onto B? Which represent a 1-1 function?

   a)
2. Which of the following figures are graphs of functions? Which of the functions are 1-1? Justify your answer.

a)

b)

c)

d)

2. Which of the following figures are graphs of functions? Which of the functions are 1-1? Justify your answer.

a)

b)

c)
3. a) Which of the following equations could be used to find angle $A$ in the diagram?
   a) $A^2 = 6.7^2 + 9.4^2$
   b) $\tan A = \frac{9.4}{6.7}$
   c) $\sin A = \frac{9.4}{6.7}$
   d) $\cos A = \frac{6.7}{9.4}$

   b) Calculate the measure of angle $A$.

4. a) The following diagram shows a ladder leaning against a house. Which of the following would you use to find the height the ladder reaches up the wall?
   a) Pythagorean Theorem
   b) Sine
   c) Cosine
   d) Tangent

   b) Calculate the height the ladder reaches up the wall.

5. Given the function $f(x) = x^2 + 2$, $x \geq 0$:
   a) Use technology to draw the graph of $f$. 
b) On the same axis, draw the graph of \( f^{-1} \).

c) Algebraically, determine the expression for \( f^{-1}(x) \).

**Revisiting \( y = c + a \cos b(x - d) \)**

**Solving Trigonometric Equations Analytically**

**Solve Equations (with double angles)**

**Solving Trigonometric Equations Using Inverse Notation**

**Quiz Two**

1. a) For the following graph, list the transformations of and write an equation to model the graph.

![Graph](image)

b) Use the equation to calculate the height at \( 115^\circ \).

2. Determine the exact value of the following trigonometric function by using the half-angle identities

   a) \( \tan \left( \frac{7\pi}{8} \right) \)

   b) \( \sin 22.5^\circ \)

3. Express the product \( 2 \sin x \sin 2x \) as a sum in terms of sine and cosine.

4. Prove that the following statement is an identity:

   \[
   \sin 3x \sin x - \cos 3x \cos x = 2
   \]

5. Use the sum-to-product formula to evaluate \( \sin 30^\circ + \sin 60^\circ \)

Note: This chapter is very technical and has an emphasis on the trigonometric identities as they relate to solving trigonometric equations. Many of the applications of solving trigonometric equations deal with topics that may not be familiar to the students. Therefore, the application problems must have explicit instructions for reaching the solution and the problem itself should not be long and involved.

**Test One**

**Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.**

1. Solve the trigonometric equation \( \sqrt{2} \sin \theta = 1 \) such that \( 0 \leq \theta < 2\pi \).

2. As a recent graduate of Roller Coaster College and an expert on sinusoidal curves, your first job was designing the wave part of a new roller coaster ride for Glace Bay. Horizontally 15 m into your wave part, you are at the top of your wave. Another 45 m horizontally will put you at the bottom of your wave. The lowest point is 3 m below ground level and vertically the distance between the highest and lowest point is
30 m.
Sketch a graph to model the wave part of the ride and write an equation to model the situation. How high are you on the roller coaster 45 m horizontally into the ride?

3. Solve the equation \(2 \cos^2 x - \sin x - 1 = 0\) over the interval \(0 \leq x < 2\pi\).

4. Solve the trigonometric equation \(\sec^2 x + 2 \tan x - 6 = 0\) such that \(0 \leq x < 2\pi\).

5. Solve the equation \(\cos 3x \cos x + \sin 3x \sin x = 1\) over the interval \(0^\circ \leq x < 360^\circ\).

6. To find the angle subtended by an object on a camera film, it is necessary to solve the equation \(\frac{p^2 \tan \theta}{0.0063 + p \tan \theta} = 1.6\), where \(p\) is the distance from the camera to the object.

Find \(\theta\) if \(p = 5.2\) ft.

7. Evaluate the following expressions and give all answers in exact values:
   a) \(\tan(\sin^{-1} 0)\)
   b) \(\cos(2 \sin^{-1} 1)\)
   c) \(\tan\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)\)

8. Find the exact value of \(x\).
   a) \(\tan^{-1} x = \sin^{-1}\left(\frac{2}{5}\right)\)
   b) \(\sec^{-1} x = -\sin^{-1}\left(-\frac{1}{2}\right)\)

9. Prove that the sum of the angles on the left side of the equation equals the sine of the angle on the right.

\[
\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \sin^{-1}\left(\frac{56}{63}\right)
\]

10. To calculate the angle between two sections of a robot arm, the equation \(1.20 \cos \theta + 0.135 \cos 2\theta = 0\) must be solved. Find the measure of \(\theta\) if \(0^\circ < \theta < 180^\circ\).

**Test Two**

**Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.**

1. A Ferris wheel at the town fair with a radius of 21 feet makes one complete revolution every 16 seconds. The bottom of the wheel is 4.5 feet above the ground.
   a) Draw a graph to illustrate how a person’s height above the ground varies with time.
   b) Write an equation to model this situation.
c) Use the equation to calculate a person’s height above the ground at 24 seconds.

2. Solve the following trigonometric equation such that \( \theta \in \mathbb{R} \).

\[
4\sin^2 \theta - 3\sin \theta + 1 = 0
\]

3. Evaluate the following and express all answers as exact values in simplest radical form.

a) \( \cos \left( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) \)

b) \( \tan \left( \arcsin \left( \frac{1}{2} \right) \right) \)

4. An equation used in astronomy is \( \theta - e \sin \theta = M \). Solve for \( \theta \) when \( e = 0.25 \) and \( M = 0.75 \).

5. Solve the equation \( \tan 2\theta - \cot 2\theta = 0 \) over the interval \([0, 2\pi)\)

6. Solve the equation \( \sin^2 \left( \frac{\theta}{2} \right) - \cos \theta + 0 = 0 \) for \( 0 \leq \theta < 2\pi \)

7. Given \( \sin \alpha = \frac{5}{13} \) and \( \sin \beta = \frac{4}{5} \) and both angles are acute, find \( \tan(\alpha - \beta) \).

8. State the amplitude, period, phase shift and equation of the sinusoidal axis for the function \( y = 3\sin 2\left( x + \frac{\pi}{6} \right) - 4 \).

9. Determine the inverse of the following trigonometric function and use technology to verify the result. Sketch the graphs of the function and of the inverse.

\[
f(x) = 4 \cos^{-1} \left( \frac{2}{x - 5} \right)
\]

10. The intensity of a certain type of polarized light is given by the equation \( I = I_0 \sin 2\theta \cos 2\theta \). Solve for \( \theta \).
Chapter 5

Teacher’s Edition Assessment: Triangles and Vectors

5.1 Teacher’s Edition Assessment: Triangles and Vectors

Pretest

1. Using the formula \( d = \sqrt{x^2 + y^2} \), calculate the distance from the origin to the given point.
   a) (3, 0)
   b) (−8, 12)
   c) (−6, −15)
   d) (8, −15)

2. Find the coordinates of each indicated point \((x, y)\)
   a)
3. Using the formula \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \), calculate the distance between the following points.
   a) (5, 3), (1, 1)
   b) (4, −8), (−5, 6)
   c) (−3, −1), (−4, −3)

4. In each of the following, determine the measure of \( \theta \) and \( l \) or \( \alpha \).
5. Solve the following equation in terms of \( \cos A \).

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

**The law of Cosines**

**Area of a Triangle**

**The Law of Sines**

**The Ambiguous Case**

**General Solutions of Triangles**

**Quiz One**

Part A – Multiple Choice – Please *circle the letter* for the correct answer and *write that letter* in the blank to the left of each question.

1. _____ What is the length of side \( a \) in \( \triangle ABC \)?
   a) \( a = 112.0 \)
   b) \( a = 10.6 \)
   c) \( a = 5.9 \)
   d) \( a = 4.2 \)

2. _____ What is the area of the \( \triangle ABC \)?
a) 62.9 cm²  
b) 68.7 cm²  
c) 31.5 cm²  
d) 34.3 cm²

3. Which of the following would give the correct value for angle $A$ in the following triangle?
   a) $A = \cos\left[\frac{81-49-25}{70}\right]$  
   b) $A = \cos^{-1}\left[\frac{81+49+25}{70}\right]$  
   c) $A = \cos^{-1}\left[\frac{81-49-25}{70}\right]$  
   d) $A = \cos^{-1}\left[\frac{81-49-25}{70}\right]$  

4. Given $\triangle ABC$, what is the length of side ‘$b$’?
5. _____ Given the following triangle, what is the measure of the smallest angle?

6. _____ If \( \sin \angle C = .6268 \), what are two values for \( \angle C \)

7. _____ Given \( \triangle ABC \) where \( \angle A = 42^\circ, a = 30 \text{ in}, b = 25 \text{ in} \), what number of solutions exist?

8. _____ What is the area of a triangle that has side lengths of 13 ft., 15 ft., and 18 ft?

a) \( b = 4.62 \text{ cm.} \)
b) \( b = 6.42 \text{ cm.} \)
c) \( b = 5.18 \text{ cm.} \)
d) \( b = 2.57 \text{ cm} \)

a) \( 87.3^\circ \)
b) \( 41.8^\circ \)
c) \( 50.9^\circ \)
d) \( 36.9^\circ \)

a) \( 141.2^\circ \) and \( 38.8^\circ \)
b) \( 51.2^\circ \) and \( 128.8^\circ \)
c) \( 32.1^\circ \) and \( 147.9^\circ \)
d) \( 62.7^\circ \) and \( 117.3^\circ \)

a) zero
b) one
c) two
d) three

a) Area \( \approx 9200 \text{ ft}^2 \)
b) Area \( \approx 48 \text{ ft}^2 \)
c) Area \( \approx 96 \text{ ft}^2 \)
d) Area \approx 4600 \text{ ft}^2

Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.

1. Calculate the measure of the largest angle in \( \triangle ABC \).

![Diagram of \( \triangle ABC \) with sides 8.1 cm, 8.4 cm, and 4.8 cm.]

2. Solve the following triangle:

![Diagram of \( \triangle ABC \) with sides labeled a=60.0, b=60.0, and angle B=30°.]

3. An eight meter telephone pole has a very bad lean and creates an angle greater than 90° with the ground. A guide wire, 14 m long is attached to the pole for support so the pole will not fall down. The guide wire is anchored in the ground at a point 10 m from the base of the pole. Calculate the angle that the pole makes with the ground.

Vectors

Component Vectors

Real World Triangle Problem Solving

Quiz Two

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _____What effect does a scalar multiple have on a vector?
   a) None
   b) It changes the direction of the vector.
   c) It increases the magnitude of the vector by the multiple.
   d) It increases the magnitude of the vector by the multiple and changes its direction.

2. _____In the following diagram, what is the value of \( R \)?
a) $6\mathbf{A}\mathbf{B}$  
b) $3\mathbf{A} + (-2\mathbf{B})$  
c) $2\mathbf{B} + (-3\mathbf{A})$  
d) $3\mathbf{A} + 2\mathbf{B}$

3. What is the magnitude of the x-component in the following diagram if $A = 6.75$?

a) $x$-component $\approx 3.17$  
b) $x$-component $\approx 5.96$  
c) $x$-component $\approx 14.38$  
d) $x$-component $\approx 7.64$

4. What is the y-component of the vector that has a magnitude of 2.65 and a direction of 197.3°?

a) $y$-component $\approx -2.53$  
b) $y$-component $\approx -0.79$  
c) $y$-component $\approx 2.53$  
d) $y$-component $\approx 0.79$

Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.

1. If the magnitude of the vector is 680, calculate the horizontal and vertical components of the vector shown in the following diagram.

2. A nuclear submarine approaches the surface of the ocean at 22 m/h at an angle of 17°18’ with the
surface. What are the components of its velocity?

3. If vectors A and B are at right angles such that \( \vec{A} \) is 14.7 and \( \vec{B} \) is 19.2, what is the direction and magnitude of the resultant vector?

4. A boat leaves a dock and travels 1580 miles due west, then turns 35° to the south and continues to travel another 1640 miles to a second dock. What is the displacement of the second dock from the first dock?

5. A grandfather clock has a pendulum 97.5 cm long. From one end of the swing to the other, the straight line separation is 18.4 cm. Through what angle does the pendulum swing?

Test One

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _____ What is the length of side a in \( \triangle ABC \) if \( b = 14 \text{ cm}, c = 11 \text{ cm} \) and \( \angle A = 786^\circ \)?
   a) \( a = 15.9 \text{ cm} \).
   b) \( a = 11.6 \text{ cm} \).
   c) 12.8 cm.
   d) 13.1 cm

2. _____ If vectors A and B are at right angles such that \( \vec{A} \) is 327 and \( \vec{B} \) is 505, what is the direction and magnitude of the resultant vector?
   a) \( R \approx 384.8 \text{ and } \theta \approx 40.4^\circ \) with \( A \)
   b) \( R \approx 601.6 \text{ and } \theta \approx 57.1^\circ \) with \( A \)
   c) \( R \approx 832 \text{ and } \theta \approx 49.6^\circ \) with \( A \)
   d) \( R \approx 178 \text{ and } \theta \approx 40.4^\circ \) with \( A \)

3. _____ Given the following triangle, what is the length of side ‘c’?

   ![Diagram of triangle ABC with sides labeled]

   a) 65 cm.
   b) 55.6 cm.
   c) 114.8 cm.
   d) 74.3 cm.

4. _____ What is the magnitude and direction of the resultant of \( \vec{A} = 780, \theta_A = 28^\circ \) and \( \vec{B} = 346, \theta_B = 320^\circ \)
a) $R \approx 688$ and $\theta \approx 68^\circ$
b) $R \approx 265$ and $\theta \approx 12^\circ$
c) $R \approx 964$ and $\theta \approx 8.6^\circ$
d) $R \approx 853$ and $\theta \approx 252^\circ$
5. If $\sin \angle C = .7456$, what are two values for $\angle C$
   a) $48.2^\circ$ and $228.2^\circ$
   b) $48.2^\circ$ and $131.8^\circ$
   c) $48.2^\circ$ and $138.2^\circ$
   d) $48.2^\circ$ and $311.8^\circ$
6. Given the following triangle, what is its area?

![Triangle Diagram]

   a) Area $\approx 189$ cm$^2$
   b) Area $\approx 378$ cm$^2$
   c) Area $\approx 283.5$ cm$^2$
   d) Area $\approx 300$ cm$^2$

Part B - Answer each of the following questions in the space provided. Show all the work necessary to answer each question. Work neatly and carefully.

1. Two hockey players hit the puck at the same time, striking it with horizontal forces of $34.5$ N and $19.5$N that are perpendicular to each other. Determine the resultant of these forces.

2. A tree on a hillside casts a shadow of $58$ meters down the hill. If the angle of elevation of the sun is $48.3^\circ$ and the angle of inclination of the hill is $21.4^\circ$, find the height of the tree.

3. An airplane is headed due east with a wind blowing from the southeast. What is the velocity of the plane with respect to the earth’s surface if the velocity of the plane is $600$ km/h and the velocity of the wind is $100$ km/h?

4. Your backyard is a great spot to build a skating rink this winter. The ground must be covered with sand that costs $16$ per square meter. If the sides of the triangular plot of land measure $8.24$ m, $7.67$ m, and $8.13$ m, what will be the cost of the sand to cover this area?

Test Two

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.
1. What is the area of a triangle that has side lengths of 9 in, 10 in, and 11 in?
   a) Area $\approx 42.4$ in$^2$
   b) Area $\approx 1800$ in$^2$
   c) Area $\approx 450$ in$^2$
   d) Area $\approx 489.3$ in$^2$

2. What is the sum of the following vectors: $\vec{B}[-2,5]$ and $\vec{C}[-3,-1]$?
   a) $\vec{B} + \vec{C} = [4,-5]$
   b) $\vec{B} + \vec{C} = [1,4]$
   c) $\vec{B} + \vec{C} = [-5,4]$
   d) $\vec{B} + \vec{C} = [5,-4]$

3. If $\triangle ABC$ has $b = 25$ ft, $c = 30$ ft and $\angle B = 27^\circ$ and the triangle is solved, how many solutions exist?
   a) One
   b) None
   c) Three
   d) Two

4. What conditions must be present in order to use the Law of Cosines?
   a) S.S.A. and S.A.S
   b) A.A.S. and S.S.S.
   c) S.S.A. and A.A.S.
   d) S.A.S and S.S.S.

5. What is the measure of $\angle A$ in the following triangle?
   a) $100^\circ$
   b) $135.9^\circ$
   c) $121.6^\circ$
   d) $315.1^\circ$

6. If vectors $A$ and $B$ are at right angles such that $\vec{A}$ is 14.7 and $\vec{B}$ is 19.2, what is the direction and magnitude of the resultant vector?
   a) $R \approx 24.2$ and $\theta \approx 52.6^\circ$ with $A$
   b) $R \approx 12.35$ and $\theta \approx 50^\circ$ with $A$
   c) $R \approx 41.2$ and $\theta \approx 40^\circ$ with $A$
   d) $R \approx 33.9$ and $\theta \approx 37.4^\circ$ with $A$

Part B - Answer each of the following questions in the space provided. Show all the work
necessary to answer each question. Work neatly and carefully.

1. A ship sails 34.5 miles due east and then turns 42.3° north of east. After sailing another 18.2 miles, where is it with reference to the starting point?

2. Calculate the horizontal and vertical components for the above forces.

3. The angle of depression of a fire noticed west of a fire tower is 6.2°. The angle of depression of a cabin, also west of the tower, is 13.5°. If the tower is 2.25km from the cabin on a direct line to the cabin, how far is the fire from the cabin?

4. An air traffic controller observes two planes that are due east from the control tower and headed toward each other. One plane is 17.2 miles from the tower at an angle of elevation of 26.4°, and the other plane is 33.6 miles from the tower at an angle of elevation of 12.4°. How far apart are the planes?

5. Solve \( \triangle ABC \)
Chapter 6

Teacher’s Edition Assessment: Polar Equations and Complex Numbers

6.1 Teacher’s Edition Assessment: Polar Equations and Complex Numbers

This chapter is quite long and presents topics that are new to students. Many of the topics involve using formulas, all of which are unknown by the students, to solve the problems. Since the students are being exposed to the concepts for the first time, a pretest for the chapter is not recommended. However, instead of two quizzes, three will be made available for this chapter. Each quiz will be designed to test the student’s comprehension of the topics. Applications of these topics will be addressed in the tests. Three tests will be presented for this chapter. These tests will be for the entire chapter but you may want to choose questions to compose smaller tests to be administered at various times during the unit. If this type of testing is chosen, then test two and test three should be cumulative. Test two would cover the new topics since test one and also include topics from the first test. Test three would cover the entire unit.

Polar Coordinates

Sinusoids of One Revolution

Graphs of Polar Equations

Rectangular to Polar

Polar Equations and Complex Numbers

Quiz One

1. Plot the following polar coordinates on a polar coordinate grid:
   a) \( (3, \frac{\pi}{6}) \)
b) $\left(0.5, \frac{8\pi}{3}\right)$
c) $(-2, 225^\circ)$

2. Determine two sets of polar coordinates for the following rectangular coordinates.
   a) $\sqrt{3}, 1$
   b) $(2\sqrt{2}, 2\sqrt{2})$
   c) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

3. Find the rectangular coordinates for each of the points for which the polar coordinates are given.
   a) $(-4, -\pi)$
   b) $\left(8, \frac{4\pi}{3}\right)$

4. Using technology, graph each of the following equations and, in detail, identify the curve.
   a) $r = 1 - 2 \sin \theta$
   b) $r = 2 \cos 2\theta$

5. Express the following rectangular equations as polar equations:
   a) $x + 2y = 3$
   b) $x^2 + 4y^2 = 4$

6. Determine the rectangular equation of the following polar equations:
   a) $r \cos \theta = 4$
   b) $r = 4 \cos \theta + 2 \sin \theta$

7. Identify each of the equations as representing either a circle, a parabola, an ellipse or a hyperbola.
   a) $2x^2 + 2y^2 - 3y - 1 = 0$
   b) $2x(x - y) = y(3 - y - 2x)$
   c) $2x^2 - y^2 - 1 = 0$
   d) $2.2x^2 - x - y = 16$

Graph and Calculate Intersection of Polar Curves

Equivalent Polar Curves

Recognize

\[ i = \sqrt{-1}, \sqrt{x}, i\sqrt{x} \]
Standard Forms of Complex Numbers

The Set of Complex Numbers

Complex Number Plane

Quadratic Formula

Quiz Two

1. Using Technology, plot the polar curves represented by the following equations:

\[ r_1 = 2 + 2 \cos \theta \] and \[ r_2 = 2 + \sin \theta \]. Using the graph, determine the points of intersection of the polar curves.

2. Of the following polar equations, which two produce graphs that are the same curve?

\[ r_1 = 2 + 3 \sin \theta \]
\[ r_2 = -2 + 3 \sin \theta \]
\[ r_3 = 2 - 3 \sin \theta \]

3. Express the following radicals in terms of \(i\):

a) \(\sqrt{-12}\)

b) \(\sqrt{-0.49}\)

c) \(\sqrt{-18}\)

4. Perform the indicated operation and express each answer in the standard form \(a + bi\).

a) \(\sqrt{28} - \sqrt{-63}\)

b) \(\sqrt{80} - \sqrt{-45}\)

c) \(\sqrt{64} + 3\sqrt{-27}\)

5. Plot each number on the complex number plane and determine the distance from the origin to each of the points.

a) \(8 - 5i\)

b) \(5 + 4i\)

6. For the following quadratic equation, describe the nature of the roots and then solve the equation to determine the exact roots.

\[ 3x^2 - 5x + 4 = 0 \]

7. What values of \(x\) and \(y\) satisfy the equation \(2x + 3iy = -6 + 12i\)?
Sums and Differences of Complex Numbers

Products and Quotients of Complex Numbers

The Trigonometric or Polar Form of a Complex Number

De Moivre’s Theorem

nth Root Theorem

Solve Equations

Quiz Three

1. Perform the indicated operations and express all answers in the form $a + bi$
   a) $(5 - 8i) + (3 - i)$
   b) $0.26 - (0.56 - 0.19i) + 0.74i$
   c) $(9 - i) - (4 - 4i) + (8 - 1)$

2. Determine the following products and express all answers in the form $a + bi$.
   a) $(7i)(7 - i)$
   b) $(5 - 5i)(6 + 7i)$
   c) $(\sqrt{-18} \sqrt{-4})(5i)$

3. What is the conjugate of each complex number?
   a) $4 - 9i$
   b) $-3 + 5i$
   c) $-7 + i$

4. Perform the indicated operations and express all answers in the form $a + bi$.
   a) $\frac{6i}{2 - i}$
   b) $\frac{6 + 5i}{3 - 4i}$
   c) $\frac{4i}{1 - i} - \frac{3 + i}{2 + 3i}$

5. Represent each complex number graphically and write its polar form.
   a) $4 + 3i$
   b) $-3.00 + 4.00i$
   c) $1 + i\sqrt{3}$

6. Represent each complex number graphically and write its rectangular form.
   a) $3(\cos 232^\circ + i \sin 232^\circ)$
   b) $1.80(\cos 150^\circ + i \sin 150^\circ)$

7. Find the product of the complex numbers $4(\cos 60^\circ + i \sin 60^\circ)$ and $2(\cos 20^\circ + i \sin 20^\circ)$ and express the answer in polar form.

8. Determine the quotient of the following and express the answer in polar form.
9. Using DeMoivre’s theorem, determine
   a) \((5 + 4i)^4\)
   b) \((2 + 3i)^5\)

10. Using DeMoivre’s theorem, find all the indicated roots.
   a) The two square roots of \(4(\cos 60° + i \sin 60°)\)
   b) The cube roots of \(3 - 4i\)

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**Test One**

Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. Plot the points \((2, \frac{5\pi}{6})\) and \((-3.6, -2.8)\) on the polar grid.

2. The following graph represents a rose whose equation is \(r = 4 \sin 2\theta\).
Express the equation in rectangular form.

3. An architect has designed a domed roof for the new arena. To complete this task, he has used the equation $x^2 + y^2 = k^2$, where $k$ is a constant. Express this equation in polar form.

4. The following graph shows the intersection of two polar curves. What are the names of each of these curves? Determine the intersection points of these curves algebraically.

5. Two forces acting on an overhead fixture can be represented by $35 - 20i$ and $-50 - 18i$. Determine the resultant force graphically and verify the result algebraically.

6. The voltage of a battery is represented by $2.84 - 1.06i$ volts. Write this voltage in polar form.

7. What are the six sixth roots of $-8$?

8. The electric power (in watts) supplied to an element in a circuit is the product of the voltage ($e$) and the current ($i$) (in amps). Determine the expression for the power supplied if $e = 7.20\angle56.3^\circ$ volts and $i = 7.45\angle-15.8^\circ$ amps.
Test Two

Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. Plot the following points on the complex number plane and then calculate the absolute value of the points.
   a) $7 + 4i$
   b) $6 - 3i$
   c) $-5 + 6i$
   d) $-3 - 7i$

2. Perform the indicated operations and express the result in simplest rectangular form.
   a) $(-3 - 2i) - (-5 - \sqrt{-49})$
   b) $(-5 + 4i)(8 - 3i)$
   c) $\frac{3 + \sqrt{-4}}{4 - 7}$

3. Two cables lift a boat. The tensions in the cables can be represented by $240 - 120i$ N and $150 + 560i$ N. Express the resultant tension in polar form.

4. Use DeMoivre”s theorem to find the fifth roots of $32i$.

5. Perform the indicated operations algebraically and check them graphically.
   a) $(-2 + 5i) + (5 + 6i)$
   b) $(7 + 2i) - (3 - 7i)$

6. For the following polar curves, determine the coordinates of the intersection points and prove the results algebraically.
7. For the equation \(2y^2 - 9x^2 = 18\) determine the vertices, the foci and sketch the curve.

8. What is the rectangular equation of the polar equation \(r = \frac{4}{2-\cos \theta}\)?

Test Three

Part A – Multiple Choice – Please circle the letter for the correct answer and write that letter in the blank to the left of each question.

1. _____What is the polar form of the rectangular equation \(y = 2x\)
   a) \(\theta = \cos^{-1}(2)\)
   b) \(\theta = \sin^{-1}\left(\frac{1}{2}\right)\)
   c) \(\theta = \tan^{-1}(2)\)
   d) \(\theta = \sec^{-1}(2)\)

2. _____What is the equation of the curve that has its center at \((1, -2)\) and passes through the point \((4, -3)\)?
   a) \((x - 1)^2 + (y - 2)^2 = 5\)
   b) \(x^2 + y^2 - 2x + 4y - 5 = 0\)
   c) \((x + 1)^2 + (y + 2)^2 = 5\)
   d) \(x^2 + y^2 + 2x - 4y - 5 = 0\)

3. _____How many real solutions exist for the following systems of equations?
   a) One
   b) Two
   c) Three
   d) Four

4. _____What is the conjugate of the complex number \(3 - 7i\)?
   a) \(-3 - 7i\)
   b) \(3 + 7i\)
c) $-3 + 7i$

d) None exists

5. _____What values of $x$ and $y$ satisfy the equation $6i - 7 = 3 - yi - x$

a) $x = 10, y = -6$

b) $x = -6, y = 10$

c) $x = 10, y = 6$

d) $x = 6, y = 10$

6. _____What is the product of $(3 - 7i)^2$?

a) $9 + 49i$

b) $9 - 14i$

c) $40 + 42i$

d) $-40 - 42i$

7. _____What is the quotient of $\frac{0.25}{3 - \sqrt{-1}}$?

a) $\frac{3}{4} + \frac{1}{4}i$

b) $0.075 + 0.025i$

c) $-0.075 - 0.025i$

d) $-\frac{3}{4} - \frac{1}{4}i$

8. _____Which operation with complex numbers does the following graph represent?

a) $(4 + 2i) + (2 + 3i)$

b) $(4 + 2i) - (2 + 3i)$

c) $(4 - 2i) + (2 - 3i)$

d) $(2 - 3i) - (4 - 2i)$

9. _____What is $4.75 \angle 172.8^\circ$ expressed in rectangular form.

a) $4.70 + .595i$
b) $0.595 - 4.70i$

c) $-4.70 + 0.595i$

d) $-4.70 - 0.595i$

10. ______ What is the specific name of the curve shown below?

[Image of a polar curve]

a) a cardioid

b) a rose

c) a dimpled limaçon

d) a limaçon with an inner loop

Part B - Answer all questions in the space provided. Show all of the work necessary to obtain the solution. If you are unable to complete an answer, do as much as you can.

1. In an alternating-current circuit, the voltage $E$ is represented by the equation $E =IZ$, where $I$ is the current in amps. and $Z$ is the resistance in ohms. If $I = 0.835 - 0.427i$ amps and $Z = 250 + 170i$ ohms calculate the voltage in the circuit.

2. Using technology, plot the graph of each of the following and name the polar curve. Be specific when you name the curve.

   a) $r = 2 - 3 \sin \theta$

   b) $r = 2 \sin 2\theta$

   c) $r = 2 + \cos \theta$

3. Determine the cube roots of $-2 + 2i$.

4. What is the trigonometric form of the complex number shown below where the argument satisfies $0 \leq \theta < 2\pi$?

[Image of a complex number]

5. For the given point $(-6, \frac{5}{6})$, determine four different pairs of polar coordinates that represent this point such that $-2\pi \leq \theta \leq 2\pi$. 

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