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Chapter 1

TE Equations and Functions - Solution Key

1.1 Complete Solutions to Even-Numbered Review Exercises

Variable Expressions

2. $1.35 \cdot y = 1.35y$
4. $\frac{1}{4} \cdot z = \frac{1}{4}z$ or $\frac{1}{4} \cdot z = \frac{1}{4}$
6. $4c + d = 4(5) + (-4) = 20 - 4 = 16$
8. $\frac{2a}{c-d} = \frac{2(-3)}{5-(-4)} = \frac{-6}{9} = \frac{-2}{3}$
10. $\frac{a-4b}{3c+2d} = \frac{(-3)-4(2)}{3(5)+2(-4)} = \frac{-3-8}{15-8} = \frac{-11}{7}$
12. $\frac{ab}{cd} = \frac{(-3)(2)}{(5)(-4)} = \frac{-6}{20} = \frac{3}{10}$
14. $\frac{5x^2}{6x^3} = \frac{5(1)^2}{6(3)^3} = \frac{5}{6 \cdot 27} = \frac{5}{162} = \frac{5}{162}$
16. $x^2 - y^2 = (-1)^2 - (2)^2 = 1 - 4 = -3$
18. $2x^2 - 3x^2 + 5x - 4 = 2(-1)^2 - 3(-1)^2 + 5(-1) - 4 = 2 \cdot 1 - 3 \cdot 1 - 5 - 4 = -10$
20. $3 + \frac{1}{z^2} = 3 + \frac{1}{(-3)^2} = 3 + \frac{1}{9} = \frac{27}{9} + \frac{1}{9} = \frac{28}{9}$
22a. When $x = 2$, $V = 4(2)(10 - 2)^2 = 8(8)^2 = 8 \cdot 64 = 512$ cubic inches.
22b. When $x = 3$, $V = 4(3)(10 - 3)^2 = 12(7)^2 = 12 \cdot 49 = 588$ cubic inches.

Order of Operations

2a. $\frac{jk}{j+k} = \frac{(6)(12)}{6+12} = \frac{72}{18} = \frac{18.4}{18.1} = \frac{4}{1} = 4$
2b. $2y^2 = 2(5)^2 = 2(25) = 50$
2c. $3x^2 + 2x + 1 = 3(5)^2 + 2(5) + 1 = 3(25) + 10 + 1 = 86$
2d. \((y^2 - x)^2 = ([1]^2 - 2)^2 = (1 - 2)^2 = (-1)^2 = 1\)

4a. \((5 - 2) \cdot (6 - 5) + 2 = 5\)

4b. \((12 \div 4) + 10 - (3 \cdot 3) + 7 = 11\) Note that the equation is true as it stands (without parentheses).

4c. \((22 - 32 - 5) \cdot (3 - 6) = 45\)

4d. \(12 - (8 - 4) \cdot 5 = -8\)

**Patterns and Equations**

2a. Let \(x\) represent the number of cookies eaten by you; let \(y\) represent the number of cookies left after you have eaten some. Then:

\[ y = 24 - x \]

2b. \(x = 9\) cookies; thus, \(y = 15\) cookies left.

4. Step 1. Extract the important information:

The following table shows some values of the related quantities.

<table>
<thead>
<tr>
<th>Liters of water</th>
<th>Liters of alcohol</th>
<th>Total liters in mixture</th>
<th>Percent alcohol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1/2 (50%)</td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>2.5</td>
<td>1/2.5 (40%)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1/3 (33.3%)</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1/4 (25%)</td>
</tr>
</tbody>
</table>

**Figure 1:** Although guessing a few values as above proved successful, an equation that describes the relationship mathematically can be derived from the table. We can see from the table that the percent of alcohol in the mixture is “the number of liters of alcohol divided by the total number of liters in the mixture.”

Step 2. Translate into a mathematical equation:

Let \(x\) represent the unknown number of liters of water.

25\%(0.25) is the percent of alcohol in the mixture.

The number of liters of alcohol is 1.

\(0.25 = \frac{1}{x + 1}\) THIS IS OUR EQUATION

Step 3. Solve for \(x\) by multiplying both sides by \((x + 1)\). This yields:

\(0.25(x + 1) = 1\)

Multiply both terms by 0.25. Then

\(0.25x + 0.25 = 1\)

Subtract 0.25 from both sides.

\(0.25x = 0.75\)

Divide both sides by 0.25.

\(x = 0.75/0.25 = 3\) liters.
6. Step 1. Extract the important information:
The item decreased in price in the last year.
The item decreased by 20% in the last year.
This year the price of the item is $120.
The unknown is the price of the item last year.
Step 2. Translate into a mathematical equation:
Let \( x \) represent the unknown price of the item last year.
“20% of the item” translates to \( 0.20x \).
The text “the item decreased by 20% in the last year” translates to the expression:
\[ x - 0.20x = 120 \] THIS IS OUR EQUATION
Step 3. Solve for \( x \) by collecting like terms. This yields:
\[ 0.80x = 120 \]
Divide both sides by 0.80.
\[ x = 120/0.80 = $150 \]
Step 4. Check the result.
20% of $150 is $30. A 20% decrease in price means $150 – $30 = $120. The answer checks out.

**Equations and Inequalities**

2a. Define: Let \( x \) = the number of seats on the bus
Translate: A bus seats 65 or fewer people

\[ x \leq 65 \]

Answer: The inequality is: \( x \leq 65 \)

2b. Define: Let \( n \) = an integer; \( n + 1 \) = the next integer after \( n \)
Translate: the sum of two consecutive integers is less than 54

\[ n + (n + 1) < 54 \]

Answer: The inequality simplifies to: \( 2n + 1 < 54 \)

2c. Define: Let \( P \) = the amount of money invested at 5% annual interest.
Translate: the interest earned = \( 0.05P \)
All together now: The interest earned at the end of the year is greater than or equal to $250.

\[ 0.05P \geq 250 \]

Answer: The inequality is: \( 0.05P \geq 250 \)

2d. Define: Let \( n \) = the number of hamburgers you can buy with your money.
Translate: you have at most $3 to spend.

\[ 0.49n \leq 3 \]
Answer: The inequality simplifies to: \(0.49n \leq 3\)

4a.
\[
2(12 + 6) \leq 8(12) \\
2(18) \leq 96 \\
36 \leq 96
\]
This is a true statement.

4b.
\[
1.4(-9) + 5.2 > 0.4(-9) \\
-12.6 + 5.2 > -3.6 \\
-7.4 > -3.6
\]
This is not a true statement.

4c.
\[
-\frac{5}{2}(40) + \frac{1}{2} < -18 \\
-100 + \frac{1}{2} < -18
\]
This is a true statement.

4d.
\[
80 \geq 10(3(0.4) + 2) \\
80 \geq 10(1.2 + 2) \\
80 \geq 10(2.2) \\
80 \geq 22
\]
This is a true statement.

6. Define: Let \(x\) = the amount of total sales.

Translate: $1000 per month plus 6% of total sales

This means: monthly salary = 0.06\(x\) + 1000

Translate: $1200 per month plus 5% of sales over $2000.

This means: monthly salary = 0.05(\(x\) − 2000) + 1200

Translate: For what amount of sales is the first option better than the second option?

This means: when is 0.06\(x\) + 1000 > 0.05(\(x\) − 2000) + 1200 ?

Simplify:
\[
0.06x + 1000 > 0.05(x - 2000) + 1200 \\
0.06x + 1000 > 0.05x - 100 + 1200 \\
0.01x > 100 \\
x > 10000
\]

Answer: You should make more than $10,000 in monthly sales if the first option is going to be better than the second option.

**Functions as Rules and Tables**

2. Domain: Maria cannot work “negative hours”; so it seems the domain should be any number greater than or equal to zero. This means that the domain of the function is all non-negative rational numbers.
Range: Right from the start, it is given that Maria will make $15. Money is similarly measured in rational units (rounded to the nearest cent in such situations). This means that the range of the function is all rational numbers greater than 15.

4. Domain: Since there are no numbers that cannot be squared, multiplied by 2 and then increased by 5, the domain of f is all real numbers.

Range: Since the $x^2$ is non-negative (negatives cancel in pairs), $x^2$ is smallest when $x = 0$. Thus, $f$ will be smallest when $x = 0$; and its value will be 5. $f$ increases as $x$ increases. All this implies that the range of $f$ is all real numbers greater than or equal to 5.

6. $\{-1, -4, -5\}$

8.

Table 1.2:

<table>
<thead>
<tr>
<th>hours</th>
<th>earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$32.50</td>
</tr>
<tr>
<td>10</td>
<td>$65</td>
</tr>
<tr>
<td>15</td>
<td>$97.50</td>
</tr>
<tr>
<td>20</td>
<td>$130</td>
</tr>
<tr>
<td>25</td>
<td>$162.50</td>
</tr>
<tr>
<td>30</td>
<td>$195</td>
</tr>
</tbody>
</table>

10.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$\sqrt{3}$</td>
</tr>
<tr>
<td>1</td>
<td>$\sqrt{5}$</td>
</tr>
<tr>
<td>2</td>
<td>$\sqrt{7}$</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>$\sqrt{11}$</td>
</tr>
<tr>
<td>5</td>
<td>$\sqrt{13}$</td>
</tr>
</tbody>
</table>

12. Comparing the first table column to the remaining ones yields a pattern:

Table 1.3:

<table>
<thead>
<tr>
<th>Hours</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$15 = 15 + 0 = 15 + 5(0)$</td>
</tr>
<tr>
<td>1</td>
<td>$20 = 15 + 5 = 15 + 5(1)$</td>
</tr>
<tr>
<td>2</td>
<td>$25 = 15 + 10 = 15 + 5(2)$</td>
</tr>
<tr>
<td>3</td>
<td>$30 = 15 + 15 = 15 + 5(3)$</td>
</tr>
</tbody>
</table>

The output of cost is 15 increased by some multiple of 5. The multiple of 5 depends on the input. Let $C = \text{cost}$ and $h = \text{hours}$. Then a function is $C(h) = 15 + 5h$. 

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Table 1.4:

<table>
<thead>
<tr>
<th>pieces</th>
<th>cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 4: Let $x$ represent the number of pieces into which a ribbon is cut. Let $f$ represent the number of cuts necessary to obtain $x$ pieces. Then a function is: $f(x) = x - 1$.

16. Define: Let $x =$ the number of bracelets Rochelle needs to make to break even.
Translate: She makes bracelets for $12.50 each.
To find the total amount she makes, we multiply $12.50 by $x$.
To find the break even point, we need to compare how much she makes and how much was initially invested.

$$2500 - 12.50x = 0$$

Solve:

$$2500 - 12.50x = 0$$
$$2500 = 12.50x$$
$$200 = x$$

Answer: Rochelle needs to make 200 bracelets before breaking even.

Functions as Graphs

2a. $(−6, 4)$
2b. $(7, 6)$
2c. $(−8, −2)$
2d. $(4, −7)$
2e. $(5, 0)$
4a. See graphs provided in the SE.
4b. See graphs provided in the SE.
4c. See graphs provided in the SE.
6a. The general shape of the graph points to the absolute value function. For each $x$–value input, the corresponding $y$–value output is half of it. So the function rule is: $y = \frac{1}{2}|x|$.
6b. The general shape of the graph points to the square root function. For each $x$–value input, the corresponding $y$–value output is exactly the square root of it. So the function rule is: $y = \sqrt{x}$.
8a. 63 years
8b. 69 years
8c. 74 years
8d. 76 years
10a. Yes, a function.
10b. No, not a function.

## Problem-Solving Plan

2. Step 1: Understand.

We know: This year you got a 5% raise.
Your new salary is $45,000.
We want: To know your salary before the raise.

Step 2: Strategy.

Let’s look at the given information and try to find the relationship between the information we know and the information we are trying to find.

A 5% raise means that your salary has increased by 5% of itself; that is, $0.05 \times (salary \ before \ the \ raise) + (salary \ before \ the \ raise)$

So, let $x$ = your salary before the raise. Then your salary after the raise can be computed as follows

$$45000 = 0.05x + x$$

Step 3: Solve.

$$45000 = 0.05x + x$$
$$45000 = 1.05x$$
$$42857.14 \approx x$$

Answer: Your salary before the raise was about $42,857.14.

4. Step 1: Understand.

We know: An employee gets a 15% discount.
A $10 coupon is applied to the purchase of a $65 purse.
We want: the final cost of the purse.

Step 2: Strategy.

Let’s look at the given information and try to find the relationship between the information we know and the information we are trying to find.

Since the employee discount is to be applied before the coupon, 15% will need to be taken off the price of the purse before subtracting the $10 coupon.

In symbols, final cost = 65 – 0.15(65) – 10

Step 3: Solve.

$$\text{final cost} = 65 - 0.15(65) - 10$$
$$\text{final cost} = 45.25$$

Answer: The employee will spend $45.25 on the purse.

Step 4: Check
The answer checks out.

We know: It costs $12 to get into the fair.
Rides cost $1.50.
Rend spent $24 total.
We want: to know how many rides Rena went on.

Step 2: Strategy.
Let’s look at the given information and try to find the relationship between the information we know and the information we are trying to find.

If Rena goes on x rides, then she will have spent 1.50x total at the fair. Putting all the givens together,

\[ 24 = 1.50x + 12 \]

Step 3: Solve.

\[ 24 = 1.50x + 12 \]
\[ 12 = 1.50x \]
\[ 8 = x \]

Answer: Rena went on 8 rides.

Step 4: Check
The answer checks out.

8. Step 1: Understand.
We know: The sum of the angles in a triangle is 180 degrees.
The second angle is twice the size of the first angle.
The third angle is three times the size of the first angle.
We want: To know the measures of the angles in the triangle.

Step 2: Strategy.
Let’s look at the given information and try to find the relationship between the information we know and the information we are trying to find.

Since the second and third angles can be calculated knowing the measure of the first angle, let \( x \) = the measure of the first angle.

Then the measure of the second angle = 2x
and the measure of the third angle = 3x

Since (measure of the first angle) + (measure of the second angle) + (measure of the third angle) = 180, we have

\[ x + (2x) + (3x) = 180 \]

Step 3: Solve.

\[ x + (2x) + (3x) = 180 \]
\[ 6x = 180 \]
\[ x = 30 \]
Then the second angle measures \(2(30) = 60\) and the third angle measures \(3(30) = 90\).

Answer: The angles measure 30, 60, and 90 degrees.

Step 4: Check
The answer checks out.

**Problem-Solving Strategies: Make a Table; Look for a Pattern**

2. Step 1: Understand.
Brit has $2.25 in nickels and dimes.
She has 40 coins in total.
We want to know how many of each coin she has.

Step 2: Strategy.
Start by making a table of the different ways Brit can have 40 coins in nickels and dimes. We can calculate the total amount of money for each case.

Look for patterns appearing in the table that can be used to find the solution.

Step 3: Apply strategy/solve.

<table>
<thead>
<tr>
<th>Nickels</th>
<th>Dimes</th>
<th>Total Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>20</td>
<td>(0.05(20) + 0.10(20) = $3)</td>
</tr>
<tr>
<td>25</td>
<td>15</td>
<td>(0.05(25) + 0.10(15) = $2.75)</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>(0.05(30) + 0.10(10) = $2.50)</td>
</tr>
</tbody>
</table>

**Figure 5:** We see that every time we reduce the number of dimes by five and increase the number of nickels by five, the total amount is decreased by $0.25. The first entry in the table gives a total amount of $3 so we have 3 quarters to go until we reach our goal. This means that we should reduce the number of dimes by 15 and increase the number of nickels by 15. We have:
5 dimes and 35 nickels, making a total of 40 coins and \(5(\$0.10) + 35(\$0.05) = \$2.25\)

Answer: Brit has 5 dimes and 35 nickels.

Step 4: Check:
Answer checks out.

4. Step 1: Understand.
Oswald starts with 24 cups the first week.
He cuts down to 21 cups the second week.
He cuts down to 18 cups the third week.
His goal is cut down to 6 cups per week.
We want to know how many weeks it will take him to reach 6 cups per week.

Step 2: Strategy.
We can make a table for weeks and number of cuts; but it is clear that Oswald is cutting down 3 cups per week. Let's write an equation.

Step 3: Apply strategy/solve:

He begins with 24 cups and cuts down on the total number by 3 each week. That is, he drinks each week 3 less cups than the previous week. Let $x =$ number of weeks since the first week. Then $3x$ represents the total number of cups cut down since the first week.

The equation to solve is: $24 - 3x = 6$

Solving the equation:

\[
\begin{align*}
24 - 3x &= 6 \\
24 - 6 &= 3x \\
18 &= 3x \\
6 &= x
\end{align*}
\]

Answer: It will take Oswald 6 weeks to reach his goal. This means that starting on the seventh week, he will be drinking his targeted amount.

Step 4: Check:

Answer checks out.


One car has been traveling at 75 mph.

Another car has been traveling at 55 mph.

The cars are traveling in the same direction.

The slower car started 2 hours before the faster.

We want to know how many hours will the faster car take to catch up to the slower car.

Step 2: Strategy.

Make a table for each car: their distances traveled and the time elapsed.

Step 3: Apply strategy/solve:

Table 1.6:

<table>
<thead>
<tr>
<th>Time Elapsed</th>
<th>Distance (slower car)</th>
<th>Distance (faster car)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 hour</td>
<td>55 miles</td>
<td>0 miles</td>
</tr>
<tr>
<td>2 hours</td>
<td>110 miles</td>
<td>0 miles</td>
</tr>
<tr>
<td>3 hours</td>
<td>165 miles</td>
<td>75 miles</td>
</tr>
<tr>
<td>4 hours</td>
<td>220 miles</td>
<td>150 miles</td>
</tr>
<tr>
<td>5 hours</td>
<td>275 miles</td>
<td>225 miles</td>
</tr>
<tr>
<td>6 hours</td>
<td>330 miles</td>
<td>300 miles</td>
</tr>
<tr>
<td>7 hours</td>
<td>385 miles</td>
<td>375 miles</td>
</tr>
<tr>
<td>8 hours</td>
<td>440 miles</td>
<td>450 miles</td>
</tr>
</tbody>
</table>

Figure 6: The table indicates the faster car surpasses the slower car between the 7th and 8th hour. To know more precisely when this happens, a similar table can be constructed with half hour, 20 – minute, or smaller increments of time; but if we want to know exactly (to the second, for example), it would be
easier to solve an equation.

The slower car is going 55 mph. If \( t \) represents the elapsed time, then \( 55t \) represents the total distance traveled by the slower car. Since the faster car left two hours later and travels at 75 mph, the distance it goes can be represented by \( 75(t - 2) \). We want to know when the cars meet – this is when their distances are the same. So we want to solve:

\[
55t = 75(t - 2)
\]

\[
55t = 75t - 150
\]

\[
-20t = -150
\]

\[
t = 7.5
\]

Answer: The faster car will surpass the slower car 5.5 hours after leaving; that is, 7.5 hours after the slower car leaves.

Step 4: Check:

8. Step 1: Understand.

The garden is to be rectangular.

Lemuel has 24 feet of fencing.

We want to know the largest possible area that he could enclose with the fence.

Step 2: Strategy.

We can make a table for the length and width of the garden and then calculate the area of the enclosed region.

We need to remember the formula for the area of a rectangle:

\[
\text{Area} = \text{width} \times \text{length}
\]

We also need to remember the formula for the perimeter of a rectangle:

\[
\text{Perimeter} = 2(\text{width}) + 2(\text{length})
\]

Step 3: Apply strategy/solve:

Beginning with the perimeter formula, we have:

\[
24 = 2(\text{width}) + 2(\text{length})
\]

\[
12 = \text{width} + \text{length}
\]

\[
\text{width} = 12 - \text{length}
\]

Table 1.7:

<table>
<thead>
<tr>
<th>length (feet)</th>
<th>width = 12 - length (feet)</th>
<th>Area = length × width (sq.feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>35</td>
</tr>
</tbody>
</table>
Table 1.7: (continued)

<table>
<thead>
<tr>
<th>length (feet)</th>
<th>width = 12 − length (feet)</th>
<th>Area = length × width (sq. feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>

**Figure 7:** Note that this problem cannot be solved exactly until gaining some knowledge of quadratic equations introduced in a later chapter. By making a new table with smaller, and therefore non-integer increments of length (0.5 feet, 0.2 feet, etc), it will be seen that the area in the last column does not surpass 36 sq. ft.

Answer: Lemuel should enclose a square plot of land, 6 feet on a side. The largest possible area that he could enclose is 36 sq. ft.

Step 4: Check:
Answer checks out.
Chapter 2

TE Real Numbers - Solution Key

2.1 Complete Solutions to Even-Numbered Review Questions

Integers and Rational Numbers

2a. There are three equal portions in the circle, and of these, one is shaded, so the shaded region is $\frac{1}{3}$ of the total.

2b. Each quarter of the circle is divided into three pieces, so there are 12 pieces in total. Of these, 7 are shaded, so the shaded region is $\frac{7}{12}$ of the total.

2c. Assume the rectangle is divided into $7 \times 10 = 70$ equal squares. Then the shaded region represents 44 squares out of 70, which is a fractional area of $\frac{44}{70}$ or reducing $\frac{22}{35}$.

4a. $\frac{22}{34} = \frac{2 \cdot 11}{2 \cdot 17} = \frac{11}{17}$

4b. $\frac{9}{27} = \frac{3 \cdot 3}{3 \cdot 9} = \frac{1}{3}$

4c. $\frac{12}{18} = \frac{2 \cdot 3}{2 \cdot 9} = \frac{2}{3}$

4d. $\frac{315}{420} = \frac{3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 7} = \frac{3}{4}$

6a. Find the absolute value first, according to the order of operations: $11 - | - 4 | = 11 - (4) = 11 - 4 = 7$

6b. Find the absolute value first, according to the order of operations: $|4 - 9| - | - 5 | = (5) - (5) = 0$

6c. Find the absolute value first, according to the order of operations: $| - 5 - 11 | = | - 16 | = 16$

6d. Find the absolute value first, according to the order of operations: $7 - |22 - 15 - 19| = 7 - | - 12 | = 7 - (12) = -5$

6e. Find the absolute value first, according to the order of operations: $- | - 7 | = -(7) = -7$

6f. Find the absolute value first, according to the order of operations: $| - 2 - 88 | - | 88 + 2 | = | - 100 | - | 100 | = (100) - (100) = 0$. 
Addition of Rational Numbers

2a. \( \frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7} \)

2b. \( \frac{3}{10} + \frac{1}{5} = \frac{3}{5} + \frac{1}{5} = \frac{3+1}{10} = \frac{4}{10} = \frac{2}{5} \)

2c. \( \frac{5}{16} + \frac{3}{4} = \frac{5}{4} + \frac{3}{4} = \frac{5+3}{4} = \frac{8}{4} = 2 \)

2d. \( \frac{8}{25} + \frac{7}{10} = \frac{8}{5} + \frac{7}{10} = \frac{8+7}{10} = \frac{15}{10} = \frac{3}{2} \)

2e. \( \frac{8}{27} + \frac{7}{18} = \frac{8}{3} + \frac{7}{18} = \frac{8+7}{3} = \frac{15}{3} = 5 \)

2f. \( \frac{1}{6} + \frac{1}{4} = \frac{1}{3} + \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{3}{3} = \frac{2+3}{6} = \frac{5}{6} \)

2g. \( \frac{5}{19} + \frac{2}{27} = \frac{5}{3} + \frac{2}{3} = \frac{5+2}{3} = \frac{7}{3} \)

2h. \( \frac{7}{19} + \frac{9}{27} = \frac{7}{3} + \frac{9}{27} = \frac{7+9}{27} = \frac{16}{27} \)

4. The fractions of the price of the ice-cream that are paid are:

\( \frac{1}{2} \) From Nadia

\( \frac{1}{3} \) From Peter

\( \frac{1}{4} \) From Ian

The total fraction that gets paid towards the price of the ice-cream is: \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \). Determine the LCD.

The LCM of 2, 3 and 4 is 12. Each fraction is equivalent to:

\[
\begin{align*}
\frac{1}{2} &= \frac{1 \cdot 6}{2 \cdot 6} = \frac{6}{12} \\
\frac{1}{3} &= \frac{1 \cdot 4}{3 \cdot 4} = \frac{4}{12} \\
\frac{1}{4} &= \frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}
\end{align*}
\]

The sum is \( \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12} \).

\( \frac{13}{12} \) is greater than 1, we know that the three friends put down more money than the price of the ice-cream; but because they put down exactly the right amount of money to pay the total cost (with tax added), the fraction of the tax must have been the fraction over 1, that is \( \frac{13}{12} - 1 = \frac{13}{12} - \frac{12}{12} = \frac{1}{12} \).

Solution: The fraction of the tax paid is \( \frac{1}{12} \).

An alternative way to obtain the same answer, using variables is as follows:

Let \( p = \) price of the ice-cream

Let \( t = \) the amount of tax on the ice-cream

The sum of the fractions that each person paid equals the total cost of the item (price plus tax).

\[
\frac{1}{2}p + \frac{1}{3}p + \frac{1}{4}p = p + t
\]

\[
\frac{1}{2}p + \frac{1}{3}p + \frac{1}{4}p - p = t
\]

—Solving for \( t \) gives:

\[
\frac{1}{12}p = t
\]
The tax paid is one-twelfth of the price of the ice-cream.
Solution: The fraction of the tax paid is $\frac{1}{12}$.

**Subtraction of Rational Numbers**

2. When $x = 3, y = 11$ and when $x = 7, y = 23$. The difference in the two $y$–values will give the change in the function from $x = 3$ to $x = 7$.

$$\text{Change} = 3(7) + 2 - [3(3) + 2] = 23 - 11 = 12$$

Solution: When the $x$–values go from 3 to 7, the $y$–values increase overall by 12.

4. When speed = 40 mph, time = 3 hrs and when speed = 90 mph, time = 4/3 hrs. The difference in these times will give the change of time from 40 mph to 90 mph.

$$\text{Change} = \frac{120}{90} - \frac{120}{40} = \frac{4}{3} - 3 = \frac{4}{3} - \frac{9}{3} = \frac{4 - 9}{3} = \frac{-5}{3}$$

Solution: By switching to a new high-speed train, a commuter travels $-\frac{5}{3}$ hrs longer than he or she would by bus; that is, they save $\frac{5}{3}$ hrs = $1\frac{2}{3}$ hrs.

**Multiplication of Rational Numbers**

2a. $\frac{5}{12} \times \frac{9}{10} = \frac{15}{120} \times \frac{9}{10} = \frac{1}{4} \times \frac{9}{2} = \frac{3}{8}$

2b. $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

2c. $\frac{3}{7} \times \frac{1}{7} = \frac{3}{49}$

2d. $\frac{15}{17} \times \frac{9}{17} = \frac{135}{289}$. Note that no reduction is possible here.

2e. $\frac{1}{13} \times \frac{1}{1} = \frac{1}{13}$

2f. $\frac{7}{27} \times \frac{9}{14} = \frac{1}{17} \times \frac{9}{14} = \frac{1}{17} \times \frac{1}{2} = \frac{1}{34}$

2g. $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$

2h. $\frac{1}{11} \times \frac{22}{1} \times \frac{7}{10} = \frac{1}{11} \times \frac{22}{1} \times \frac{7}{10} = \frac{1}{11} \times \frac{17}{2} \times \frac{1}{5} = \frac{1}{55}$

2i. $\frac{12}{11} \times \frac{10}{13} \times \frac{9}{2} = \frac{12}{11} \times \frac{10}{13} \times \frac{9}{2} = \frac{12}{11} \times \frac{10}{13} \times \frac{9}{2} = \frac{1}{3} \times \frac{10}{13} \times \frac{9}{2} = \frac{70}{9}$

**The Distributive Property**

2a. $\frac{1}{2}(x - y) - 4 = \frac{1}{2}x - \frac{1}{2}y - 4 = \frac{x}{2} - \frac{y}{2} - 4$

2b. $0.6(0.2x + 0.7) = (0.6)(0.2x) + (0.6)(0.7) = 0.12x + 0.42$

2c. $6 + (x - 5) + 7 = 6 + x - 5 + 7 = x + (6 - 5 + 7) = x + 8$

2d. $6 - (x - 5) + 7 = 6 - x + 5 + 7 = 18 - x$

2e. $4(m + 7) - 6(4 - m) = 4m + 28 - 24 + 6m = 10m + 4$

2f. $-5(y - 11) + 2y = (-5)(y) + (-5)(-11) + 2y = 55 - 5y + 2y = 55 - 3y$

4. Each shelf has (7 poetry + 11 novels) books. Since there are 5 shelves, the total number of books is $5(7 \text{ poetry} + 11 \text{ novels})$.
By the distributive property, \(5(7 \text{ poetry} + 11 \text{ novels}) = 5(7 \text{ poetry}) + 5(11 \text{ novels}) = 35 \text{ poetry} + 55 \text{ novels}\).

Solution: The bookshelf has a total of 35 poetry books and 55 novels.

**Division of Rational Numbers**

2a. \(\frac{5}{2} ÷ \frac{1}{4} = \frac{5}{2} × \frac{4}{1} = \frac{5}{1} × \frac{2}{1} = 10\)

2b. \(\frac{1}{2} ÷ \frac{7}{9} = \frac{1}{2} × \frac{9}{7} = \frac{9}{14}\)

2c. \(\frac{5}{11} ÷ \frac{6}{7} = \frac{5}{11} × \frac{7}{6} = \frac{35}{66}\)

2d. 1, since anything (not 0) divided by itself is 1. Another way to see this is using a reciprocal: \(\frac{1}{2} ÷ \frac{1}{2} = \frac{1}{2} × \frac{2}{1} = 1\)

2e. \(-\frac{x}{2} ÷ \frac{5}{7} = -\frac{x}{2} × \frac{7}{5} = -\frac{7x}{10}\)

2f. \(\frac{1}{2} ÷ \frac{x}{4y} = \frac{1}{2} × \frac{4y}{x} = \frac{4y}{2x} = \frac{2y}{x}\)

2g. \((-\frac{1}{3}) ÷ (-\frac{3}{8}) = (-\frac{1}{3}) × (-\frac{8}{3}) = \frac{5}{9}\)

2h. \(\frac{7}{2} ÷ \frac{7}{4} = \frac{7}{2} × \frac{4}{7} = \frac{28}{14} = 2\)

2i. \(11 ÷ (-\frac{4}{5}) = 11 × (-\frac{5}{4}) = -\frac{55}{4}\)

4. To determine the time, we need the distance covered, and the speed at which it moves. Note that our numbers involve all the right units to answer the question.

\[
speed = \frac{3}{8} \text{ mph}\\
distance = \frac{2}{3} \text{ mile}\\
time =?\]

The values are then substituted for the corresponding variables into a formula relating all three quantities: distance, speed and time. The formula provided in the text (see Examples 3 and 4) is:

\[
speed = \frac{distance}{time}\]

Since we are looking for time, it would be helpful to solve this equation for time first to avoid messy arithmetic. By multiplying both sides of the above equation by time, we obtain

\[
distance = speed × time\]

and now dividing both sides by speed yields:

\[
time = \frac{distance}{speed}\]

The values are now substituted for the corresponding variables into this formula:

\[
time = \frac{2}{3} ÷ \frac{3}{8} = \frac{2}{3} × \frac{8}{3} = \frac{16}{9}\]

Solution: The digger will take \(\frac{16}{9}\) hrs = \(1(7/9)\) hrs to complete the trench.
Square Roots and Real Numbers

2. Students will plug the square roots into a calculator to find the given answers.

4. Using a calculator to find decimal expansions for each of the four numbers, we have:

\[
\frac{\sqrt{6}}{2} \approx 1.225, \quad \frac{61}{50} = 1.220, \quad \sqrt{1.5} = \frac{\sqrt{6}}{2}, \quad \frac{16}{13} \approx 1.230
\]

It is necessary to find the decimal expansions at least out to the third decimal place. The two numbers containing radicals are equal; no calculator display should ever convince you of this fact, but it appears to be true and you’ll learn how to prove it in a later chapter. Thus, from lowest to highest, we have:

\[
\frac{61}{50} = 1.220, \quad \sqrt{1.5} = \frac{\sqrt{6}}{2} \approx 1.225, \quad \frac{16}{13} \approx 1.230
\]

Problem-Solving Strategies: Guess and Check, Work Backward

2. Step 1: Understand:

Nadia and Peter are 6 miles apart.

Nadia’s speed is 3.5 mph

Peter’s speed is 6 mph

We want to know WHEN they meet AND HOW far from home is their meeting place. Notice that the answer to the second question is equal to the distance Nadia has traveled – since she is traveling from home.

Step 2: Strategy:

Nadia’s speed is 3.5 mph

Peter’s speed is 6 mph

Guess the number of hours they travel and use this guess to calculate both of their distances traveled.

Keep guessing until the sum of their distances is 6 miles.

Step 3: Apply strategy/solve:

Guess: 1 hour

Check: (3.5 mph)(1 hr) = 3.5 miles and (6 mph)(1 hr) = 6 miles

Then sum is 3.5 + 6 = 9.5 (too large)

Guess: 1/2 hour

Check: (3.5 mph)(0.5 hr) = 1.75 miles and (6 mph)(0.5 hr) = 3 miles

Then sum is 1.75 + 3 = 4.75 (too small)

Guess: 3/4 hour

Check: (3.5 mph)(0.75 hr) = 2.625 miles and (6 mph)(0.75 hr) = 4.5 miles

Then sum is 2.625 + 4.5 = 7.125 (too large)

At this point, it might seem worthwhile to convert all the givens into minutes and use minutes instead hoping to see a clearer pattern. Instead, keep narrowing down on the answer.

Guess: 0.60 hr(3/5 ths of an hour or 36 minutes).
Check: \((3.5 \text{ mph})(0.60 \text{ hr}) = 2.1 \text{ miles}\) and \((6 \text{ mph})(0.60 \text{ hr}) = 3.6 \text{ miles}\)
Then sum is \(2.1 + 3.6 = 5.7\) (close)
By “zooming in” further, you can obtain as many correct decimal places as you have time for!
Answer: Nadia and Peter each have to travel approximately \(0.632 \text{ hrs}(37.92 \text{ minutes})\) before meeting. They will be \((3.5 \text{ mph})(0.632 \text{ hrs}) = 2.212 \text{ miles}\) away from home.
Step 4: Check:
Nadia: \((3.5 \text{ mph})(0.632 \text{ hr}) + (6 \text{ mph})(0.632 \text{ hr}) = 6.004 \text{ miles}\) (close enough!)
The answer checks out.
4. Step 1: Understand:
Andrew has 22 coins.
His coins can only be dimes and quarters.
The total value of his coins is $4.
We want to know HOW MANY of each kind of coin he has.
Step 2: Strategy:
Andrew has 22 coins in some combination of dimes and quarters.
The coins value $4 in total.
Guess the number of quarters he has, subtract this from 22 to find the number of dimes, and calculate the total value of the coins.
Keep guessing until the total value is $4.
Step 3: Apply strategy/solve:
We know that 16 quarters are worth $4, so Andrew must have less than 16 quarters. We can use this observation to decide which number to guess first.
Guess: 10 quarters, 12 dimes
Check: \((10)(0.25) + (12)(0.10) = 2.50 + 1.20 = 3.70\) (too small)
Guess: 11 quarters, 11 dimes
Check: \((11)(0.25) + (11)(0.10) = 2.75 + 1.10 = 3.85\) (too small, but just barely)
Guess: 12 quarters, 10 dimes
Check: \((12)(0.25) + (10)(0.10) = 3.00 + 1.00 = 4.00\) (correct!)
Step 4: Check:
The answer checks out.
6. Step 1: Understand:
Peter sees 13 heads and 26 feet.
There are only pigs and chickens in the yard.
A pig has one head and four feet, while a chicken has one head and two feet.
We want to know HOW MANY of each kind of animal Peter sees.
Step 2: Strategy:
Peter sees 13 heads and 36 feet.
Each animal contributes one head, but chickens contribute two feet while pigs contribute four.

Guess the number of chickens (or pigs), subtract this from 13 to find the number of the other kind of animal, and calculate the total number of feet.

Keep guessing until there are 26 feet.

Step 3: Apply strategy/solve:

We know that 7 pigs have 28 feet, so there must be less than 7 pigs. We can use this observation to decide which number to guess first.

Guess: 6 pigs, 7 chickens

Check: (6)(4) + (7)(2) = 24 + 14 = 38 feet (too large)

Guess: 5 pigs, 8 chickens

Check: (5)(4) + (8)(2) = 20 + 16 = 36 feet (correct!)

Step 4: Check:
The answer checks out.

8. Step 1: Understand:

There are 16 candies left in the bowl.

Nadia, Peter, Andrew, and Anne each took various portions of the candy left in the bowl when they found it.

We want to know HOW MANY candies were originally in the bowl.

Step 2: Strategy:

There are 16 candies left in the bowl.

Work backwards, starting with 16 candies, to find how many candies were in the bowl before Anne took any. Then find how many candies were there before Andrew took any, then Peter, then Nadia. This last number will be the number of candies originally in the bowl.

Step 3: Apply strategy/solve:

After Anne takes one third of the bowl, there are 16 candies. Thus 16 is 2/3 of the number of candies Anne found in the bowl. 16 = (2/3)(24), so there were 24 candies in the bowl when Anne arrived.

Andrew takes one fifth of the bowl, so 24 is 4/5 of the number of candies he found. 24 = (4/5)(30), so there were 30 candies in the bowl when Andrew arrived.

Similarly, 30 = (3/4)(40), so there were 40 candies when Peter arrived. 40 = (5/6)(48), so there were originally 48 candies in the bowl.

Step 4: Check:


The answer checks out.
Chapter 3

TE Equations of Lines - Solution Key

3.1 Complete Solutions to Even-Numbered Review Questions

One-Step Equations

2a.

\[ q - 13 = -13 \]
\[ +13 \quad +13 \]
\[ q = 0 \]

2b.

\[ z + 1.1 = 3.0001 \]
\[ -1.1 \quad -1.1 \]
\[ z = 1.9001 \]

2c.

\[ \frac{21s}{21} = 3 \]
\[ s = \frac{3}{21} \]
\[ s = \frac{1}{7} \]

2d.

\[ t + \frac{1}{2} = \frac{1}{3} \]
\[ -\frac{1}{2} \quad -\frac{1}{2} \]
\[ t = -\frac{1}{6} \]
2e.

\[
\begin{align*}
\frac{7f}{11} &= \frac{7}{11} \\
\frac{11}{7} \cdot \frac{7f}{11} &= \frac{11}{7} \cdot \frac{7}{11} \\
\frac{11}{7} \cdot \frac{f}{11} &= \frac{11}{7} \cdot \frac{7}{11} \\
\frac{f}{7} &= 1
\end{align*}
\]

2f.

\[
\begin{align*}
\frac{3}{4} &= \frac{-1}{2} \cdot y \\
\left(-\frac{2}{1}\right) \cdot \frac{3}{4} &= \left(-\frac{2}{1}\right) \cdot \left(-\frac{1}{2}\right) \cdot y \\
\frac{-3}{2} &= y
\end{align*}
\]

2g.

\[
\begin{align*}
6r &= \frac{3}{8} \\
\frac{1}{6} \cdot 6r &= \frac{1}{6} \cdot \frac{3}{8} \\
r &= \frac{1}{16}
\end{align*}
\]

2h.

\[
\begin{align*}
\frac{9b}{16} &= \frac{3}{8} \\
\frac{9b}{16} \times \frac{16}{9} &= \frac{3}{8} \times \frac{16}{9} \\
b &= \frac{2}{3}
\end{align*}
\]

4a. We know: Juan wants to sell the cake for 3 times the cost of making it. His costs are $8.50 and $1.25.

Let \( u \) represent the amount of money that he sells the cake for. We have:

\[ u = 3(8.50 + 1.25) \]

4b. The amount of money he charges for each slice can be determined by dividing the amount he sells the entire cake for by 12. Let \( v \) represent the amount of money he charges for each slice.

\[ v = \frac{u}{12} \quad \text{or} \quad 12v = u \]

4c. Profit = Amount made from selling – any costs.

Let \( w \) represent the total profit he makes on the cake. We have:

\[ w = u - (8.50 + 1.25) = 2(8.50 + 1.25) \]

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Two-Step Equations

2. Unknown: Let $x$ = the number of miles she can travel with her money
The taxi fare consists of two parts: a $2.35 hire charge and $0.75 per mile. The hire charge is a flat fee, and independent of $x$. The per-mile part depends on $x$. Lets look at how this works algebraically:
translate: $2.35$ as a hire charge plus an additional $0.75$ per mile
Total taxi fare $= 2.35 + 0.75x$
Lastly we look at the final piece of information: the amount Jade has, $10$. So our equation is:
$10 = 2.35 + 0.75x$ THIS IS OUR EQUATION
To solve for $x$, isolate $x$ first by subtracting $2.35$ from both sides; then divide both sides by $0.75$.
$\frac{10 - 2.35}{0.75} = x$
$10.2 = x$
Solution: The number of miles Jade can go is $10.2$ miles.

4. If $a \neq 0$, then the equation can always be solved for $x$.

$$ax + b = c$$
$$ax = c - b$$
$$x = \frac{c - b}{a}$$

This is the general solution to a linear equation.

Multi-Step Equations

2. The unknown quantity is the number of bags the engineer can put on the platform at once. This is $x$. The platform, at maximum, will weigh:
$$200 + 40x$$
and this must equal the $250 + 250$ kg (the strength/tension in the two steel cables).
$200 + 40x = 250 + 250$
$200 + 40x = 500$ to isolate $x$, start by combining like terms
$40x = 300$ next subtract 200 from both sides
$40x = 300$ divide by 40
$x = 30/4 = 7.5$
Answer: The engineer can put no more than 7 bags on the platform.

4. The unknown quantity is the amount Lydia inherited.
What we know: She split the inheritance into 5 equal amounts: $x/5$
She invested 3 portions of the money in a high interest bank account which added $10\%$ to the value:
$$3(x/5) + 0.10(3(x/5))$$
She invested the remaining two portions of her inheritance plus $500 in the stock market but lost $20\%$ on that money: 
$$2(x/5) + 500) - 0.20(2(x/5) + 500)$$
The accounts ended up having the same amount of money in them:

\[ 3(x/5) + 0.10(3(x/5)) = (2(x/5) + 500) - 0.20(2(x/5) + 500) \]

THIS IS OUR EQUATION

Take care of the parentheses first and distribute:

\[ 3x/5 + 0.30x/5 = 2x/5 + 500 - 0.40x/5 - 100 \]

Multiply both sides by 5 to clear the fractions:

\[ 3x + 0.30x = 2x + 2500 - 0.40x - 500 \]

Combine like terms on each side:

\[ 3.3x = 1.6x + 2000 \]

Isolate the \( x \):

\[ 1.7x = 2000 \]

Divide by 1.7:

\[ x = 1176.47 \]

Answer: Lydia inherited $1176.47.

**Equations with Variables on Both Sides**

2. The unknown in the problem is Andrew’s number.

Tamar’s trick can be written algebraically as: \( 5x - 3 \)

Manoj’s trick can be written algebraically as: \( 3(x + 5) \) [Assume Andrew begins with the same number.]

Andrew says that whichever way he does the trick he gets the same answer:

\[ 5x - 3 = 3(x + 5) \]

THIS IS OUR EQUATION

Distribute first:

\[ 5x - 3 = 3x + 15 \]

Collect like terms:

\[ 2x = 18 \]

Divide:

\[ x = 9 \]

Answer: Andrew’s original number was 9.

4. The unknown is the resistance in each component. This is \( x \).

A 2.3 Amp current flowed.
Two components were replaced with 10Ω resistors.
The current dropped to 1.9 Amps.
The formula to use here is Ohm’s Law: \( V = I \cdot R \)
Substituting into the formula yields:

\[
V = 2.3 \cdot (x + x + x + x) \quad \text{and} \quad V = 1.9 \cdot (10 + 10 + x + x + x)
\]

We know the voltage is fixed, so the \( V \) in the first equation must equal the \( V \) in the second. This means that:

\[
2.3(5x) = 1.9(20 + 3x)
\]

Multiply and distribute:

\[
11.5x = 38 + 5.7x
\]

Isolate \( x \):

\[
5.8x = 38
\]

Divide:

\[
x = 190/29
\]

Answer: Rounding to two decimal places, the resistance is 6.55Ω.

**Ratios and Proportions**

2a. \( \frac{54 \text{ hot dogs}}{12 \text{ minutes}} = 4.5 \text{ hot dogs per minute} \)
2b. \( \frac{5000 \text{ lbs}}{250 \text{ in}^2} = 20 \text{ lbs per in}^2 \)
2c. \( \frac{20 \text{ computers}}{80 \text{ students}} = 0.25 \text{ computers per student} \)
2d. \( \frac{180 \text{ students}}{6 \text{ teachers}} = 30 \text{ students per teacher} \)
2e. \( \frac{12 \text{ meters}}{4 \text{ floors}} = 3 \text{ meters per floor} \)
2f. \( \frac{18 \text{ minutes}}{15 \text{ appointments}} = 1.2 \text{ minutes per appointment} \)

4. The fixed ratio in this case will be the $908 to 100 people. The unknown, \( x \), is the amount taken in for serving 250 people.

We can go straight to the proportion:

\[
\frac{908}{100} = \frac{x}{250}
\]

Cross multiply:

\[
908 \cdot 250 = 100 \cdot x
\]

Simplify and divide:
\[ 2270 = x \]

Solution: The restaurant will make $2,270 if it were to serve 250 people.

6. What we know: 2 out of every 3 students have a cell phone.
1 in 5 of all students have a “new” cell phone.

We want to know the proportion of students owning a “new” phone out of the students who own a cell phone. That is, we need to simplify:

\[
\frac{1}{5} \cdot \frac{3}{2} = \frac{3}{10}
\]

Answer: The desired proportion is 3/10 or 30%.

**Scale and Indirect Measurement**

2a. Students will measure the length of the pool to be 3.6 cm on the diagram. According to the formula 
\((\text{distance in real life}) \times (\text{scale}) = \text{distance on diagram}\), the pool is \(\frac{500 \text{ ft}}{3 \text{ cm}} \times 3.6 \text{ cm} = 600 \text{ ft long}.

2b. Students will measure the height of the lodge to be 1.5 cm on the diagram. According to the formula 
\((\text{distance in real life}) \times (\text{scale}) = \text{distance on diagram}\), the lodge is \(\frac{500 \text{ ft}}{3 \text{ cm}} \times 1.5 \text{ cm} = 250 \text{ ft high}.

2c. The formula used is: 
\((\text{distance in real life}) \times (\text{scale}) = \text{distance on diagram}\)
Therefore, a 50 ft pool would measure:

\[
50 \text{ ft} \cdot \frac{3 \text{ cm}}{500 \text{ ft}} = \frac{3}{10} \text{ cm}
\]

Answer: The 50 ft pool would measure 0.3 cm on the map.

2d. The formula used is: 
\((\text{distance in real life}) \times (\text{scale}) = \text{distance on diagram}\)
Therefore, a 29 ft tree would measure:

\[
20 \text{ ft} \cdot \frac{3 \text{ cm}}{500 \text{ ft}} = \frac{3}{25} \text{ cm}
\]

Answer: The 20 ft tree would measure 0.12 cm on the map.

4. We could draw a scale diagram; we would see that we have two right triangles. The angle that the sun causes the shadow from the Empire State building to fall is the same angle that the yardstick shadow falls. We have 2 similar triangles, so we can say that the ratio of the corresponding sides is the same:

\[
\frac{\text{Height of Empire State building}}{\text{Length of the building’s shadow}} = \frac{\text{Length of yardstick}}{\text{Length of the yardstick’s shadow}}
\]

First, we must make certain that all the units are the same; we’ve decided to convert to feet. In addition, the length of the yardstick’s shadow needs to be changed from mixed number to decimal; 1 foot and 5 1/4 inches = 1 foot and 5.25 inches = 1 foot and 5.25/12 feet = 1.4375 feet.

\[
\frac{x}{600} = \frac{3}{1.4375}
\]

Multiply both sides by 600.

\[
x = 1252.17
\]

Answer: According to these measurements, the Empire State building is 1252.17 feet high.
Percent Problems

2a. \( \frac{1}{6} \times 100\% = 16.67\% \)
2b. \( \frac{5}{24} \times 100\% = 20.83\% \)
2c. \( \frac{6}{7} \times 100\% = 85.71\% \)
2d. \( \frac{11}{7} \times 100\% = 157.14\% \)
2e. \( \frac{13}{97} \times 100\% = 13.40\% \) (answer in SE has an extraneous minus sign)

4a. Use the percent equation. We are looking for the part. The total is 90. “of” means multiply. \( R\% \) is 30% so the rate is \( (30/100) \) or 0.30.

\[ 0.30(90) = 27 \]

Solution: 30% of 90 is 27.

4b. Use the percent equation. We are looking for the part. The total is 199. “of” means multiply. \( R\% \) is 16.7% so the rate is \( (16.7/100) \) or 0.167.

\[ 0.167(199) = 33.233 \]

Solution: 16.7% of 199 is 33.233.

4c. Use the percent equation. We are looking for the part. The total is 10.01. “of” means multiply. \( R\% \) is 11.5% so the rate is \( (11.5/100) \) or 0.115.

\[ 0.115(10.01) = 1.15115 \]

Solution: 11.5% of 10.01 is 1.15115 (answer in SE has an extraneous minus sign).

4d. Use the percent equation. We are looking for the part. The total is 3x. “of” means multiply. \( R\% \) is \( y\% \) so the rate is \( (y/100) \).

\[ (y/100)(3x) = 3xy/100 \]

Solution: \( y\% \) of 3x is 3xy/100.

6. Use the formula:

\[ \text{Percent change} = \left(1 - \frac{\text{final amount - original amount}}{\text{original amount}}\right) \times 100\% \]

Let \( x \) = pay raise

\[ 12\% = \left(\frac{x - 9.50}{9.50}\right) \times 100\% \]

Solve for \( x \): Start by dividing both sides by 100%.

\[ 0.12 = \frac{x - 9.50}{9.50} \]

Cross-multiply and add 9.50 to both sides:
Answer: The employee will be making $10.64.

8. Let \( x \) represent the supplier’s price of a particular bike.

Consider the stores’ pricing schemes:

Store A: 40\% mark-up. In symbols, \( x + 0.40x \)

Store B: 250\% mark-up and a 60\% discount. In symbols, the 250\% mark-up can be translated as \( (x + 2.5x) \). Then the discount: \( (x + 2.5x) - 0.60(x + 2.5x) \).

Simplify each expression:

Store A: 1.4x

Store B: \( (x + 2.5x) - 0.60(x + 2.5x) = x + 2.5x - 0.60x - 1.5x = 1.4x \)

Answer: The expressions are identical. This means that the stores have the same pricing schemes.

Problem-Solving Strategies: Use a Formula

2. Step one: collect relevant information.

A 500 sheet stack of paper is 1.75 inches.

The number of sheets in a 2 feet high stack of paper. = unknown = \( x \)

Step two: make an equation.

We can set up a proportion, if we convert the units to a common unit. 2 feet = 2(12) = 24 inches.

\[
\frac{500 \text{ sheets}}{1.75 \text{ inches}} = \frac{x \text{ sheets}}{24 \text{ inches}} \quad \text{THIS IS OUR EQUATION}
\]

Step three: solve.

Multiply both sides by 24.

Answer: \( x = 6857.14 \); so round down to 6857 sheets.

Step four: check your answer.

The result checks out.

4. Step one: collect relevant information.

The speed of sound in air is 340 m/s.

The guests sit 20 m from the speakers.

The time delay between picture and sound = unknown = \( x \)

Step two: make an equation.

We can use the formula  distance = (rate)(time).

Assume the picture from the screen travels to the audience instantly. Then the time delay we seek is the time it takes for the sound to travel 20 m.

\( 20 = 340x \) THIS IS OUR EQUATION

Step three: solve.

Divide both sides by 340.

Answer: \( x = 2/34 \) which is approximately 0.06 seconds (rounded to two decimal places).

Step four: check your answer.
The result checks out.

Data is stored between a radius of 2.3 cm and 5.7 cm.
The area between these two radii = unknown = x

Step two: make an equation.
The area between two concentric circles (circles having the same center) can be found by calculating the area of the smaller circle and subtracting it from the area of the larger circle.
The necessary formula is the area of a circle: \( A = \pi r^2 \).
\[ x = \pi (5.7)^2 - \pi (2.3)^2 \text{ THIS IS OUR EQUATION} \]

Step three: solve.
Answer: \( x = 85.45 \text{ sq cm} \)

Step four: check your answer.
The result checks out.
Chapter 4

TE Graphs of Equations and Functions - Solution Key

4.1 Complete Solutions to Even-Numbered Review Questions

The Coordinate Plane

2a-d.

A is in quadrant I, B is in quadrant II, C is in quadrant IV, and D is in quadrant III.

4a. Either problem-solving strategy will help: *Use a table or Look for a Pattern*. As suggested in the problem, let \( x \) = the number of Starburst that Jaeyun gives Becky and let \( y \) = the number of M&Ms Jaeyun gets in return. Then a simple table might look like the following:
The pattern is clear from the table: Becky trades three times as many of her candies as Jaeyun does.
Answer: $y = 3x$.

4b. A larger table confirms our answer in part a.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

4c.

Graphs of Linear Equations

2. “think of a number” = unknown = $x$.
   “triple it” = $3x$
   “then subtract 7” = $3x - 7$

Let this final expression be named $y$. Then $y = 3x - 7$.  

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A table of values might look like:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-7</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

4. Since there is an up-front $5 fee, this amount needs to be subtracted from your total first. An equation would look like:

$$\text{euros} = 0.7(\text{dollars} - 5)$$

Let $x$ represent the total amount of dollars you have for exchange and $y$ be the resulting amount of euros you get in exchange.

Then $y = 0.7(x - 5)$.

A table of values might look like:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.7</td>
</tr>
<tr>
<td>7</td>
<td>1.4</td>
</tr>
<tr>
<td>8</td>
<td>2.1</td>
</tr>
</tbody>
</table>

To use a graph to approximate the number of Euros you would get for $50, find $x = 50$ along the $x$–axis and go up to find the corresponding $y$–value on the line.

Answer: you would receive 31.50 Euros (exactly).

**Graphing Using Intercepts**

2a. Solve for the $y$–intercept ($x = 0$); cover up the $x$ term:

$$\Box - 6y = 15$$
$$y = -1.5$$

$(0, -2.5)$ is the $y$–intercept

Solve for the $x$–intercept ($y = 0$); cover up the $y$ term:

$$5x - \Box = 15$$
$$x = 3$$

$(3, 0)$ is the $x$–intercept

2b. Solve for the $y$–intercept ($x = 0$); cover up the $x$ term:
\[ -4y = -5 \]
\[ y = 1.25 \]

(0, 1.25) is the y–intercept
Solve for the x–intercept (y = 0); cover up the y term:
\[ 3x - \_ = -5 \]
\[ x = -5/3 \]

(-5/3, 0) is the x–intercept
2c. Solve for the y–intercept (x = 0); cover up the x term:
\[ \_ + 7y = -11 \]
\[ y = -\frac{11}{7} \]

\(0, -\frac{11}{7}\) is the y–intercept
Solve for the x–intercept (y = 0); cover up the y term:
\[ 2x + \_ = -11 \]
\[ x = -\frac{11}{2} \]

\(-\frac{11}{2}, 0\) is the x–intercept
2d. Solve for the y–intercept (x = 0); cover up the x term:
\[ \_ + 10y = 25 \]
\[ y = 2.5 \]

(0, 2.5) is the y–intercept
Solve for the x–intercept (y = 0); cover up the y term:
\[ 5x + \_ = 25 \]
\[ x = 5 \]

(5, 0) is the x–intercept
4. What we know: strawberries cost $3.00 per pound; bananas cost $1.00 per pound. Exactly $10 is going to be spent on a combination of both fruits.

The unknowns are: \(x =\) number of pounds of strawberries purchased and \(y =\) number of pounds of bananas.
Then the amount of money spent on strawberries is 3x and the amount spent on bananas is 1y.
So the total cost of the fruit is: 3x + y
Since the total amount of money to be spent is exactly $10, the following equation holds:
\[ 3x + y = 10 \]
Solve for the \( y \)-intercept \((x = 0)\); cover up the \( x \) term:

\[
\Box + y = 10 \\
y = 10
\]

\((0, 10)\) is the \( y \)-intercept

Solve for the \( x \)-intercept \((y = 0)\); cover up the \( y \) term:

\[
3x + \Box = 10 \\
x = \frac{10}{3}
\]

\((\frac{10}{3}, 0)\) is the \( x \)-intercept

A coordinate system containing the line through these two intercepts will have its \( x \)-axis labeled “strawberries” and its \( y \)-axis labeled “bananas.”

6. Expanding the parentheses using the Distributive Property, we have \(3x + 6 = 2y + 6\), or \(3x = 2y\). Using, for example, the cover-up method to find the intercepts, we find that \((0, 0)\) is both the \( x \)-intercept and the \( y \)-intercept. Since one point is not enough to determine a line, we need to find another point on the line in order to graph it.

**Slope and Rate of Change**

2a. The indicated points are \((3, 6)\) and \((-1, -6)\). slope \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 6}{-1 - 3} = \frac{-12}{-4} = 3\)

2b. The indicated points are \((0, 1)\) and \((-6, -2)\). slope \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 1}{-6 - 0} = \frac{-3}{-6} = \frac{1}{2}\)

2c. The indicated points are \((-1, 6)\) and \((5, -6)\). slope \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 6}{5 - (-1)} = \frac{-12}{6} = -2\)

2d. The indicated points are \((4, 2)\) and \((-2, -4)\). slope \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{-2 - 4} = \frac{6}{6} = 1\)

2e. The indicated points are \((4, 2)\) and \((4, -6)\). slope \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 2}{4 - 4} = -8\) undefined slope

2f. The indicated points are \((3, 1)\) and \((-6, -2)\). slope \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 1}{-6 - 3} = \frac{-3}{-9} = \frac{1}{3}\)

**Graphs Using Slope-Intercept Form**

2a. \(\frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2\)

2b. \(\frac{\Delta y}{\Delta x} = \frac{4}{3}\)

2c. \(\frac{\Delta y}{\Delta x} = \frac{0}{4} = 0\)

2d. \(\frac{\Delta y}{\Delta x} = \frac{2}{5}\)

2e. \(\frac{\Delta y}{\Delta x} = \frac{-2}{8} = -\frac{1}{4}\)

2f. \(\frac{\Delta y}{\Delta x} = \frac{-2}{4} = -\frac{1}{2}\)

2g. \(\frac{\Delta y}{\Delta x} = \frac{4}{4} = 1\)

4a-d.
Direct Variation Models

2. Assume that Dasan will continue to play games at the same rate as he did in the first 10 minutes. Note that this problem can be solved in several ways.

Let \( x \) = the elapsed time (the number of minutes played) and \( y \) = the amount spent.

\( x = 10, y = 3.50 \) We can use this information to find \( k \).

\[
y = kx \\
3.50 = k(10) \\
0.35 = k
\]

Therefore: \( y = 0.35x \) THIS IS OUR EQUATION.

To find the time when Dasan’s $20 runs out, solve:

\[
20 = 0.35x \\
57.14 \approx x
\]

Solution: Dasan can play for no more than 57 minutes and 8 seconds with the allowance he has been given.

4. The volume of the water depends on time, use \( y = kx \).

Let \( x \) = the elapsed time (the number of hours the hose is left on) and \( y \) = the volume of the water in the pool.

\( x = 8, y = 4/7 \). We can use this information to find \( k \).

\[
y = kx \\
4 = k(8) \\
1/14 = k
\]
Therefore: $y = \frac{1}{14} x$ THIS IS OUR EQUATION.

Finally, to find the time when the pool is full (100% or $y = 1$), solve:

$$1 = \frac{1}{14} x$$
$$14 = x$$

Solution: The pool will fill 14 hours after 10 PM, that is, 12:00 PM (noon) the following day.

6a. The force $F$ needed to stretch a spring by a distance $x$ is given by the equation

$$F = kx$$

where $k$ is the spring constant (measured in Newtons per centimeter, $N/cm$). $x = 10, F = 12$ use this information to find $k$.

$$F = kx$$
$$12 = k(10)$$
$$6/5 = k$$

Answer: The spring constant is $6/5$ N/cm.

6b. Now that we have the spring constant from part a, we can substitute the 7 for $x$ in the equation:

$$F = \frac{6}{5} x$$
$$F = \frac{6}{5} \cdot 7 = \frac{42}{5}$$

Answer: The force needed to stretch the spring by 7 cm is 8.4 N.

6c. Using the spring constant from part a, we can substitute 23 for $F$ in the equation:

$$F = \frac{6}{5} x$$
$$23 = \frac{6}{5} x$$
$$\frac{115}{6} = x$$

Answer: The distance the spring would stretch with a 23 N force is approximately 19.17 cm.

**Linear Function Graphs**

2. What we know: a phone card has $20 worth of calls on it. Calls cost $0.16 per minute. Each minute spent decreases the value of the card by $0.16.

$$V(x) = 20 - 0.16x$$ (in dollars)

where $V$ represents the value of the card (in dollars) after $x$ minutes of calls were made.

To find the total number of minutes you can make with the card, set $V(x) = 0$ and solve for $x$

$$0 = 20 - 0.16x$$
$$125 = x$$
Answer: The caller has 125 minutes on the phone card.

4. Both choices a) and d) pass the vertical line test and are functions. Choices b) and c) do not pass the vertical line test and are not functions.

6a. Two consecutive terms were given, so we know the common difference is $17 - (-11) = 28$. The third term is thus $17 + 28 = 45$.

To check that the third term is correct, we compute the fourth term as $45 + 28 = 73$. This matches the fourth term that was given.

Answer: the third term is 45.

6b. Since two consecutive terms were not given, the common difference is unknown. Let $d =$ common difference. Then the second term $= 2 + d$.

And then the third term $= \text{second term} + d = (2 + d) + d = -4$.

THIS IS OUR EQUATION.

Solving for $d$,

\[
\begin{align*}
(2 + d) + d &= -4 \\
2 + 2d &= -4 \\
2d &= -6 \\
d &= -3
\end{align*}
\]

The common difference is $-3$ and hence the second term is $2 + (-3) = -1$.

Answer: the second term is $-1$.

6c. Since two consecutive terms were not given, the common difference is unknown. Let $d =$ common difference.

The second term $= 13 + d$

The third term $= \text{second term} + d = (13 + d) + d = 13 + 2d$

The fourth term $= \text{third term} + d = (13 + 2d) + d = 13 + 3d$

The fifth term $= \text{fourth term} + d = (13 + 3d) + d = 0$ THIS IS OUR EQUATION.

Solving for $d$,

\[
\begin{align*}
(13 + 3d) + d &= 0 \\
13 + 4d &= 0 \\
4d &= -13 \\
d &= -13/4
\end{align*}
\]

The common difference is $-13/4$ and hence the missing terms are:

The second term $= 13 + (-13/4) = -39/4 = 9.75$

The third term $= 13 + 2(-13/4) = -26/4 = 6.5$

The fourth term $= 13 + 3(-13/4) = -13/4 = 3.25$

Answer: The missing terms are: $9.75, 6.5, 3.25$. 

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Problem-Solving Strategies - Graphs

Step 1: The candle is burning at a linear rate. After 2 minutes, the candle measures 5 inches. After 8 minutes it measures 3 inches.

We are looking for the initial length of the candle.

Let \( y \) = the length of the candle and \( x \) = the number of elapsed minutes.

Step 2: Write an equation that gives the length of the candle as a function of time.

After 2 minutes, the candle measures 5 inches. This gives point \((x, y) = (2, 5)\). After 8 minutes it measures 3 inches. This gives point \((x, y) = (8, 3)\).

The candle is burning at a linear rate, this means that the graph of the function is a line. We can find the slope of the line joining the two points and use it to write the final equation.

Step 3: Carry out the plan.

The slope of the line is: 
\[
m = \frac{5 - 3}{2 - 8} = \frac{2}{-6} = -\frac{1}{3}
\]

The equation of a line in slope-intercept form is:
\[
y = mx + b
\]

Substituting the slope:
\[
y = -\frac{1}{3}x + b
\]

When \( x = 2, y = 5 \). This allows us to solve for \( b \).

\[
5 = -\frac{1}{3}(2) + b
\]
\[
5 + \frac{2}{3} = b
\]
\[
\frac{17}{3} = b
\]

The function is
\[
y = -\frac{1}{3}x + \frac{17}{3}
\]

To answer the question of how long the candle was initially, set \( x = 0 \) in the equation.

As expected, this is the \( y \)-intercept.

Answer: The candle was originally \( \frac{17}{3} \approx 5.67 \) inches.

Step 4.

The result checks out.

Step 1: A glass of lemonade costs 45 cents. In order to break even Bobby and Petra must sell $25 worth of lemonade.

We want the number of glasses of lemonade they must sell to break even.

Let \( x \) = the number of lemonade glasses sold.

Step 2: Write an equation that gives the revenue as a function of glasses sold.
Convert cents to dollars 45 cents = $0.45.

0.45x is the amount of money they bring in. We want to know the break-even point when \(25 - 0.45x = 0\). THIS IS OUR EQUATION.

Step 3: Solve the equation:

\[
\begin{align*}
25 - 0.45x &= 0 \\
25 &= 0.45x \\
\frac{25}{0.45} &= x
\end{align*}
\]

Divide both sides by 0.45:

\[
\frac{500}{9} = x
\]

Answer: They must sell 56 glasses of lemonade (rounded up).

Step 4.

The result checks out.
Chapter 5

TE Writing Linear Equations - Solution Key

5.1 Complete Solutions to Even-Numbered Review Questions

Linear Equations in Slope-Intercept Form

2. Since we are given the slope \((m = -5)\) and \(y\)-intercept \((b = 6)\) directly, we can plug these into slope-intercept form:

\[
y = mx + b
\]
\[
y = -5x + 6
\]

4. The four steps used to find the equation of the line in slope-intercept form given the slope and a point on the line are:

(i) Start with the slope–intercept form of the line

\[
y = mx + b
\]

(ii) Plug in the given value of \(m\) into the equation

\[
y = \frac{2}{3}x + b
\]

(iii) Plug the \(x\) and \(y\) values of the given point \((\frac{1}{2}, 1)\) and solve for \(b\)

\[
1 = \frac{2}{3} \cdot \frac{1}{2} + b
\]
\[
\frac{2}{3} = b
\]

(iv) Plug the value of \(b\) into the equation

Answer: \(y = \frac{2}{3}x + \frac{2}{3}\)
6. The five steps used to find the equation of the line in slope-intercept form given the two points are:

(i) Start with the slope–intercept form of the line

\[ y = mx + b \]

(ii) Use the two points to find the slope using the slope formula:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = -2 \]

(iii) Plug in the given value of \( m \) into the equation

\[ y = -2x + b \]

(iv) Plug the \( x \) and \( y \) values of one of the given points and solve for \( b \). Using \( (5, 0) \),

\[ 0 = -2(5) + b \]
\[ 0 = -10 + b \]
\[ 10 = b \]

(v) Plug the value of \( b \) into the equation

\[ y = -2x + 10 \]

(vi) Plug the other point into the equation to check the values of \( m \) and \( b \):

\[ 6 = -2(2) + 10 \]

Answer: \( y = -2x + 10 \)

8. The five steps used to find the equation of the line in slope-intercept form given the two points are:

(i) Start with the slope–intercept form of the line

\[ y = mx + b \]

(ii) Use the two points to find the slope using the slope formula:

\[ m = \frac{5 - 0}{3 - (-3)} = \frac{5}{6} \]

(iii) Plug in the given value of \( m \) into the equation

\[ y = \left(\frac{5}{6}\right)x + b \]

(iv) Plug the \( x \) and \( y \) values of one of the given points and solve for \( b \). Using \( (-3, 0) \),

\[ 0 = \left(\frac{5}{6}\right)(-3) + b \]
\[ 0 = -5/2 + b \]
\[ 5/2 = b \]
(v) Plug the value of $b$ into the equation

$$y = (5/6)x + 5/2$$

(vi) Plug the other point into the equation to check the values of $m$ and $b$:

$$5 = (5/6)(3) + 5/2$$

Answer: $y = (5/6)x + 5/2$

10. The line passes through $(3, 0)$ and $(0, 3)$ is the y–intercept. The four steps used to find the equation of the line in slope-intercept form given the y–intercept and a point on the line are:

(i) Start with the slope–intercept form of the line

$$y = mx + b$$

(ii) Plug in the given value of $b$ into the equation

$$y = mx + 3$$

(iii) Plug in the $x$ and $y$ values of the point $(3, 0)$ and solve for $m$

$$0 = m(3) + 3$$

$$-1 = m$$

(iv) Plug the value of $b$ into the equation

Answer: $y = -x + 3$

12. Since $f(0)$ is just the y–intercept of the graph (it is the point on the graph where $x = 0$), this problem is equivalent to being given the slope and the y–intercept of the line, and we can just plug into slope-intercept form: $f(x) = 5x - 3$.

14. We are told that $m = 1/3$ and the line contains point $(-1, 2/3)$.

Start with slope–intercept form:

$$f(x) = mx + b$$

Plug in the value of the slope:

$$f(x) = \frac{1}{3}x + b$$

Plug in the point $(-1, 2/3)$

$$f(-1) = 2/3 = \frac{1}{3}(-1) + b$$

$$1 = b$$

Plug the value of $b$ in the equation:

$$f(x) = \frac{1}{3}x + 1$$
Answer: \( f(x) = \frac{1}{3}x + 1 \).

16. We are told that the line contains the points \( \left( \frac{1}{4}, \frac{3}{4} \right) \) and \( (0, \frac{5}{4}) \).

Start with slope–intercept form:

\[
f(x) = mx + b
\]

Find the slope:
\[
\frac{\frac{3}{4} - \frac{3}{4}}{0 - \frac{1}{4}} = \frac{\frac{3}{4}}{-\frac{1}{4}} = -2
\]

Plug in the value of the slope:

\[
f(x) = -2x + b
\]

Plug in the point \( (0, \frac{5}{4}) \), which is the \( y \)–intercept:

\[
f(x) = -2x + \frac{5}{4}
\]

Answer: \( f(x) = -2x + \frac{5}{4} \)

18. Let’s define our variables:

\( y \) = the amount of money Andrew has paid in dollars
\( x \) = number of months elapsed since buying the car

You can see that the problem gives us the slope of the equation and the \( y \)–intercept of the line:

- We are told that Andrew pays at a rate of 350 dollars per month, so \( m = 350 \).
- We are told that Andrew has paid a down payment of $1500, so we have the point: \( (0, 1500) \).

Start with the slope–intercept form of the line:

\[
y = mx + b
\]

Plug in the slope:

\[
y = 350x + b
\]

Plug in the \( y \)–intercept \( (0, 1500) \):

\[
y = 350x + 1500
\]

To answer the question, plug in \( x = 12 \) to obtain \( y = 350(12) + 1500 = 5700 \) dollars.

Solution: Andrew has paid a total of $5700 since buying the car.

20. Define the variables:

\( y \) = the stretched length of the spring in meters
\( x \) = the weight attached to the spring in pounds

You can see that the problem gives us two points on the line:

- We are told that for a weight of 160 lbs the spring stretches to 5 m, so we have the point \( (160, 5) \)
• We are told that for a weight of 40 lbs the cord stretches to 2 m, so we have the point (40, 2)

Start with the slope–intercept form of the line:

\[ y = mx + b \]

Find the slope of the line:

\[ \frac{5 - 2}{160 - 40} = \frac{3}{120} = \frac{1}{40} \]

Plug in the value of the slope:

\[ y = \frac{1}{40}x + b \]

Plug point (40, 2) into the equation:

\[ 2 = \frac{1}{40}(40) + b \]
\[ 1 = b \]

Plug the value of \( b \) into the equation:

\[ y = \frac{1}{40}x + 1 \]

To answer the question, we plug in \( x = 140 \) and obtain 4.5m.

Solution: The equation describing the spring is \( y = \frac{1}{40}x + 1 \). Amardeep’s weight will stretch the spring to 4.5 meters.

**Linear Equations in Point-Slope Form**

2. Start with the equation in point-slope form:

\[ y - y_0 = m(x - x_0) \]

Plug in the value of the slope, –75

\[ y - y_0 = 75(x - x_0) \]

Plug in 0 for \( x_0 \) and 125 for \( y_0 \)

\[ y - 125 = -75(x - 0) \]

Answer: \( y - 125 = -75x \)

4. Start with the equation in point-slope form:

\[ y - y_0 = m(x - x_0) \]

Find the slope using the slope formula:

\[ \frac{-2 - 3}{-1 - (-2)} = \frac{-5}{1} = -5 \]
Plug in the value of the slope, \(-5\)

\[ y - y_0 = -5(x - x_0) \]

Plug in \(-2\) for \(x_0\) and \(3\) for \(y_0\)

\[ y - 3 = -5(x - (-2)) \]

Answer: \(y - 3 = -5(x + 2)\)

Note that a different equation would have resulted if the point \((-1, -2)\) had been used instead.

6. Start with the equation in point-slope form:

\[ y - y_0 = m(x - x_0) \]

Find the slope using the slope formula:

\[ \frac{3 - 3}{2 - 0} = \frac{0}{2} = 0 \]

Plug in the value of the slope, \(0\)

\[ y - y_0 = 0(x - x_0) \]
\[ y - y_0 = 0 \]

Plug in \(3\) for \(y_0\)

\[ y - 3 = 0 \]

Answer: \(y - 3 = 0\)

8. Start with the equation in point-slope form:

\[ y - y_0 = m(x - x_0) \]

Plug in the value of the slope, \(-6\)

\[ y - y_0 = -6(x - x_0) \]

Plug in \(0\) for \(x_0\) and \(0.5\) for \(y_0\)

\[ y - 0.5 = -6(x - 0) \]

Answer: \(y - 0.5 = -6x\)

10. Here we are given the slope = \(-12\) and the point on the line gives \(x_0 = -2, f(x_0) = 5\).

Start with the equation in point-slope form:

\[ f(x) - f(x_0) = m(x - x_0) \]

Plug in the value of the slope:
\[ f(x) - f(x_0) = -12(x - x_0) \]

Plug in \(-2\) for \(x_0\) and \(5\) for \(f(x_0)\)

\[ f(x) - 5 = -12(x - (-2)) \]

Answer: \(f(x) - 5 = -12(x + 2)\)

12. Here we are given two points: \((6, 0)\) and \((0, 6)\)

Start with the equation in point-slope form:

\[ f(x) - f(x_0) = m(x - x_0) \]

Find the value of the slope:

\[
\frac{6 - 0}{0 - 6} = \frac{6}{-6} = -1
\]

Plug in the value of the slope, \(-1\)

\[ f(x) - f(x_0) = -(x - x_0) \]

Plug in 0 for \(x_0\) and 6 for \(f(x_0)\)

\[ f(x) - 6 = -(x - 0) \]

Answer: \(f(x) - 6 = -x\)

Note that a different equation would have resulted if the point \((6, 0)\) would have been used instead.

14. Here we are given the slope \(\frac{-9}{5}\) and the point on the line gives \(x_0 = 0, f(x_0) = 32\).

Start with the equation in point-slope form:

\[ f(x) - f(x_0) = m(x - x_0) \]

Plug in the value of the slope:

\[ f(x) - f(x_0) = -\frac{9}{5}(x - x_0) \]

Plug in 0 for \(x_0\) and 32 for \(f(x_0)\)

\[ f(x) - 32 = -\frac{9}{5}(x - 0) \]

Answer: \(f(x) - 32 = -\frac{9}{5}x\)

16. Since depth depends on time let's define our variables as:

\(x = \) time in minutes
\ny = depth in feet

We are given the y–intercept \((0, 400)\) and the point \((20, 50)\). Start with the equation in point-slope form:
\[ y - y_0 = m(x - x_0) \]

Find the value of the slope:
\[
\frac{400 - 50}{0 - 20} = \frac{350}{-20} = -17.5
\]

Plug in the value of the slope, \(-17.5\)
\[ y - y_0 = -17.5(x - x_0) \]

Plug in 0 for \(x_0\) and 400 for \(y_0\)
\[ y - 400 = -17.5(x - 0) \]

To answer the question, plug in \(x = 5\) and solve:
\[
\begin{align*}
y - 400 &= -17.5(5) \\
y &= 312.5
\end{align*}
\]

Answer: The equation that describes the sub’s depth as a function of time is \(y - 400 = -17.5x\). After 5 minutes, the sub will be 312.5 feet from the surface.

**Linear Equations in Standard Form**

2. We are given:
\[ y - 7 = -5(x - 12) \]

Distribute:
\[ y - 7 = -5x + 60 \]

Bring the \(x\) term to the left and the constant to the right:
\[ 5x + y = 67 \]

4. We are given:
\[ y = \frac{9}{4}x + \frac{1}{4} \]

Multiply both sides by the common denominator to clear the fractions:
\[ 4y = 9x + 1 \]

Bring the \(x\) term to the left:
As a final step, multiply both sides by 1:

\[-9x + 4y = -1\]

6. We are given:

\[3y + 5 = 4(x - 9)\]

Distribute:

\[3y + 5 = 4x - 36\]

Bring the \(x\) term to the left and the constant to the right:

\[-4x + 3y = -41\]

As a final step, multiply both sides by -1:

\[4x - 3y = 41\]

8. Because the given equation is in standard form, the slope is given by the formula \(-\frac{a}{b}\); and the \(y\)-intercept as \(\frac{c}{b}\).

Therefore, \(m = -\frac{3}{6} = -\frac{1}{2}\) and the \(y\)-intercept is \(\frac{25}{6}\).

10. Because the given equation is in standard form, the slope is given by the formula \(-\frac{a}{b}\); and the \(y\)-intercept as \(\frac{c}{b}\).

Therefore, \(m = \frac{3}{4}\) and the \(y\)-intercept is \(-\frac{20}{7}\).

12. Because the given equation is in standard form, the slope is given by the formula \(-\frac{a}{b}\); and the \(y\)-intercept as \(\frac{c}{b}\).

Therefore, \(m = -\frac{6}{1} = -6\) and the \(y\)-intercept is \(\frac{3}{1} = 3\).

14. We see that the \(x\)-intercept is approximately \((-5, 0)\) and the \(y\)-intercept is \((0, -6)\).

We saw that in standard form \(ax + by = c\), if we “cover up” the \(y\) term, we get \(ax = c\). If we “cover up” the \(x\) term, we get \(by = c\). We need to find the numbers that multiply the intercepts and give the same answer in both cases.

In this case, we see that multiplying \(x = -5\) by 6 and multiplying \(y = -6\) by 5 gives the same result:

\((x = -5) \times 6 = -30\)

and

\((y = -6) \times (5) = -30\)

Therefore, \(a = 6, b = 5\) and \(c = -30\) and the standard form is:
$$6x + 5y = -30$$

16. We see that the $x$–intercept is approximately $(-8, 0)$ and the $y$–intercept is $(0, 3)$.

We saw that in standard form $ax + by = c$, if we “cover up” the $y$ term, we get $ax = c$. If we “cover up” the $x$ term, we get $by = c$. We need to find the numbers that multiply the intercepts and give the same answer in both cases.

In this case, we see that multiplying $x = -8$ by $3$ and multiplying $y = 3$ by $-8$ gives the same result:

$$(x = -8) \times 3 = -24$$

and

$$(y = 3) \times (-8) = -24$$

Therefore, $a = 3, b = -8$ and $c = -24$ and the standard form is:

$$3x - 8y = -24$$

18. Let’s define our variables:

$x = \text{amount of money invested in the account with 5\% return.}$

$y = \text{amount of money invested in the account with 7\% return.}$

The equation that describes this situation is: $0.05x + 0.07y \leq 400$. We can rewrite this inequality without decimals by multiplying both sides by $100$. Therefore:

$$5x + 7y \leq 40000$$

If Anne invests $5000 in the 5\% account, then we plug $x = 5000$ in the equation and solve for $y$:

$$5(5000) + 7y \leq 40000$$
$$25000 + 7y \leq 40000$$
$$7y \leq 15000$$
$$y \leq 15000/7$$

Anne can invest no more than $2142.85$.

**Equations of Parallel and Perpendicular Lines**

2. Find the slope of each line and compare them.

$$m_1 = \frac{0 - (-3)}{-8 - 4} = \frac{3}{-12} = \frac{-1}{4} \quad \text{and} \quad m_2 = \frac{6 - (-1)}{-2 - (-1)} = \frac{7}{3}$$

The slopes are neither equal nor opposite reciprocals, so the lines are neither parallel nor perpendicular.

4. Find the slope of each line and compare them.
\[ m_1 = \frac{-3 - 3}{-6 - 3} = \frac{-6}{-9} = \frac{2}{3} \quad \text{and} \quad m_2 = \frac{4 - (-8)}{-6 - 2} = \frac{12}{-8} = -\frac{3}{2} \]

The slopes are negative reciprocals of each other, so the lines are perpendicular.

6. Write each equation in slope-intercept form:

Line 1: \( 5y + 3x + 1 = 0 \)

\[
5y = -3x - 1 \\
y = \frac{-3}{5}x - \frac{1}{5}
\]

Slope of line 1: \( m_1 = -\frac{3}{5} \)

Line 2: \( 6y + 10x = -3 \)

\[
6y = -10x - 3 \\
y = \frac{-10}{6}x - \frac{3}{6} \\
y = \frac{-5}{3}x - \frac{1}{2}
\]

Slope of line 2: \( m_2 = -\frac{5}{3} \)

The slopes are not the same or negative reciprocals of each other, so the lines are neither parallel nor perpendicular.

8. Find the slope of the given line:

\[ 5x - 2y = 2 \]

\[
2y = 5x - 2 \\
y = \frac{5}{2}x - 1
\]

The line has a slope of \( m_1 = \frac{5}{2} \). The slope of the parallel line is the same: \( m_2 = \frac{5}{2} \). Therefore, we are trying to find the equation of a line with slope \( m_2 = \frac{5}{2} \) that passes through point \((3, -2)\). Use the point-slope form with the slope and the point:

\[
y - y_1 = m(x - x_1) \\
y - (-2) = \frac{5}{2}(x - 3)
\]

The equation of the line could also be written as: \( y = \frac{5}{2}x - \frac{19}{2} \).

10. Find the slope of the given line:

\[ 7y + 2x - 10 = 0 \]

\[
7y = -2x + 10 \\
y = \frac{-2}{7}x + \frac{10}{7}
\]
The line has a slope of \( m_2 = \frac{-2}{7} \). The slope of the parallel line is the same: \( m_2 = \frac{-2}{7} \). Therefore, we are trying to find the equation of a line with slope \( m_2 = \frac{-2}{7} \) that passes through point \((2, 2)\). Use the point-slope form with the slope and the point:

\[
y - y_1 = m(x - x_1)
\]

\[
y - 2 = \frac{-2}{7}(x - 2)
\]

The equation of the line could also be written as:

\[
y = \frac{-2}{7}x + \frac{18}{7}
\]

12. All lines passing through the point \((0, 4)\) have the same \(y\)-intercept, \(b = 4\). The family of lines is: \(y = mx + 4\)

14. First we need to find the slope of the given line, \( y - 3 = 4x + 2 \) in slope-intercept form:

\[
y = 4x + 5
\]

The slope is 4.

All the lines in the family have a slope of \(m = 4\) but different \(y\)-intercepts. The family of lines is:

\[
y = 4x + b
\]

Fitting a Line to Data

2. Answers will vary.

Two suggested points to choose are \((39, 48)\) and \((47, 56)\). Start with the equation in point-slope form:

\[
y - y_0 = m(x - x_0)
\]

Find the value of the slope:

\[
\frac{56 - 48}{47 - 39} = \frac{8}{8} = 1
\]

Plug in the value of the slope, 1

\[
y - y_0 = x - x_0
\]

Plug in 39 for \(x_0\) and 48 for \(y_0\)

\[
y - 48 = x - 39
\]

Answer: \(y = x + 9\)

Note that a different equation would have resulted if another pair of points would have been used instead. The line of best fit is approximately \(y = 0.922x + 12.574\).

4. \(y = 2x + 6\) is an approximate best-fit line (the answer from linear regression is \(y = 1.85x + 6.49\))
6. \( y = 0.96x + 10.83 \)

8. Students should perform a linear regression for the given data points and find the equation \( y = 2.4x + 28.1 \) (note the SE gives the incorrect equation \( y = 2.5x + 27.5 \)), where \( x \) is the number of days since Shiva started training, and \( y \) is the number of samosas he can eat in 12 minutes. The \( y \)-intercept represents the number of samosas he could eat before he started training, and the slope represents the number of extra samosas per day Shiva can eat. When \( x = 14, y = 61.7 \) (note the SE solution says 57.5 here, corresponding to \( x = 12 \) in the incorrect equation), so Shiva will break the record if he continues at his current pace.

10. The line of best fit here is \( y = 1.882x + 52.85 \), where \( x \) is the number of years since 1995 and \( y \) is the
median income in thousands of dollars (note the SE gives the incorrect equation $y = 1.75x + 53.8$). The slope represents the change in median income per year, and the $y$–intercept represents the median income in 1995 (when $x$ is 0). The median income in 2010 will be, according to the line of best fit, $81,081 (the SE gives the incorrect value $80,050$).

![Median California income since 1995](image)

Predicting with Linear Models

2. Looking in the column named Median age of females, the data we need are: (1980, 22.0) and (1990, 23.9). Start with the equation in point-slope form:

$$y - y_0 = m(x - x_0)$$

Find the value of the slope:

$$\frac{23.9 - 22}{1990 - 1980} = \frac{1.9}{10} = 0.19$$

Plug in the value of the slope, 0.19

$$y - y_0 = 0.19(x - x_0)$$

Plug in 1980 for $x_0$ and 22 for $y_0$

$$y - 22 = 0.19(x - 1980)$$

Simplifying:

$$y = 0.19x - 354.2$$

To answer the question, plug in 1984 into the equation:

$$y = 0.19(1984) - 354.2$$

$$y = 22.76$$

Answer: An estimate for the median female age at first marriage in 1984 is 22.8 years.

www.ck12.org
4. The data we need are: (1996, 13.6) and (2000, 12.2). Start with the equation in point-slope form:

\[ y - y_0 = m(x - x_0) \]

Find the value of the slope:

\[
\frac{13.6 - 12.2}{1996 - 2000} = \frac{1.4}{-4} = -0.35
\]

Plug in the value of the slope, \(-0.35\)

\[ y - y_0 = -0.35(x - x_0) \]

Plug in 1996 for \(x_0\) and 13.6 for \(y_0\)

\[ y = 13.6 = -0.35(x - 1996) \]

Simplifying:

\[ y = -0.35x + 712.2 \]

To answer the question, plug in 1997 into the equation:

\[ y = -0.35(1997) + 712.2 \]
\[ y = 13.25 \]

Answer: An estimate for the percent of pregnant smokers in 1997 is 13.25%.

6. The data we need are: (1922, 12.8) and (1925, 12.4). Start with the equation in point-slope form:

\[ y - y_0 = m(x - x_0) \]

Find the value of the slope:

\[
\frac{12.4 - 12.8}{1925 - 1922} = \frac{-0.4}{3} \approx -0.1333
\]

Plug in the value of the slope, \(-0.1333\)

\[ y - y_0 = -0.1333(x - x_0) \]

Plug in 1922 for \(x_0\) and 12.8 for \(y_0\)

\[ y - 12.8 = -0.1333(x - 1922) \]

Simplifying:

\[ y = -0.1333x + 269.07 \]

To answer the question, plug in 1920 into the equation:
\[ y = -0.1333(1920) + 269.07 \]
\[ y = 13.1 \]

Answer: An estimate for the winning time for the female 100 meter race in 1920 is 13.1 seconds.

8. If we blindly apply linear extrapolation to the leftmost two data points (namely, (10.25, 60) and (10, 61)), we have a slope of \( \frac{61-60}{10-10.25} = -4 \), and an equation (from point-slope form) of \( y - 61 = -4(x - 10) \), or \( y = -4x + 101 \). For \( x = 9 \) hours of daylight, this gives \( y = 65^\circ F \). This is clearly unreasonable because the data show a general trend of increasing temperatures for increasing hours of daylight; in other words, the overall slope is positive, but the leftmost two points happen to give a negative slope, which throws off our prediction. Using the line of best fit, we find \( y = 3.196x + 29.69 \), so when \( x = 9 \) hours, \( y = 58.5^\circ F \) (to one decimal place).

![High temperature vs. daylight hours graph](image)

**Problem Solving Strategies: Use a Linear Model**

2. With the aid of a graphing calculator, the line of best fit can be calculated as

\[ y = 0.24x - 401. \]

To answer the question, we plug in 1955 for \( x \)

\[ y = 0.24(1955) - 401 = 68.2 \]

Answer: An estimate for the life expectancy of a person born in 1955 is 68.2 years.

4. With the aid of a graphing calculator, the line of best fit can be calculated as

\[ y = 0.24x - 401. \]

To answer the question, we plug in 1976 for \( x \)

\[ y = 0.24(1976) - 401 = 73.24 \]

Answer: An estimate for the life expectancy of a person born in 1976 is 73.24 years.
6. With the aid of a graphing calculator, the line of best fit can be calculated as 

\[ y = 0.24x - 401. \]

To answer the question, we plug in 2012 for \( x \)

\[ y = 0.24(2012) - 401 = 81.88 \]

Answer: An estimate for the life expectancy of a person born in 2012 is 81.88 years.

8. Because the data are very close to linear, the line of best fit gives better estimates.

10. With the aid of a graphing calculator, the line of best fit can be calculated as 

\[ y = 1.042x + 65.06. \]

To answer the question, we plug in 4.5 for \( x \)

\[ y = 1.042(4.5) + 65.06 = 69.749 \]

Answer: An estimate for the temperature in the middle of the 4\textsuperscript{th} month is 69.75 F.

12. With the aid of a graphing calculator, the line of best fit can be calculated as 

\[ y = 1.042x + 65.06. \]

To answer the question, we plug in 13 for \( x \)

\[ y = 1.042(13) + 65.06 = 78.606 \]

Answer: An estimate for the temperature in January 2007 is 78.606 F.

14. In this case, the data are highly nonlinear, so an estimate using the line of best fit would lie very far from the data points. Linear interpolation gives the best estimate because the value of the function between any two data points changes relatively little. Linear extrapolation is slightly less accurate because we don’t know anything about the behavior of the data outside the given points, but it is still better than the line of best fit.
Chapter 6

TE Graphing Linear Inequalities - Solution Key

6.1 Complete Solutions to Even-Numbered Review Questions

Inequalities Using Addition and Subtraction

2. The graph shows all numbers less than \(-10\); the open circle means \(-10\) is not included. So the inequality is \(x < -10\).

4. The graph shows all numbers greater than \(30\); the open circle means \(30\) is not included. So the inequality is \(x > 30\).

6. \(x + 15 \geq -60\)

Subtract 15 from both sides of the inequality:

\(x + 15 - 15 \geq -60 - 15\)

Simplify to obtain:

\(x \geq -75\)

8. \(x - 8 > -20\)

Add 8 to both sides of the inequality:
\[ x - 8 + 8 > -20 + 8 \]

Simplify to obtain:
\[ x > -12 \]

14. To solve the inequality:
\[ x + 65 < 100 \]

Subtract 65 from both sides of the inequality:
\[ x + 65 - 65 < 100 - 65 \]

Simplify to obtain:
\[ x < 35 \]

16. To solve the inequality:
\[ x + 68 \geq 75 \]

Subtract 68 from both sides of the inequality:
\[ x + 68 - 68 \geq 75 - 68 \]

Simplify to obtain:
\[ x \geq 7 \]

**Inequalities Using Multiplication and Division**

2. Original problem:
\[ \frac{x}{5} > -\frac{3}{10} \]

Multiply both sides by 5:
\[ 5 \cdot \frac{x}{5} > -\frac{3}{10} \cdot 5 \] (direction of inequality is NOT changed)

Simplify:
\[ x > -\frac{3}{2} \]

4. Original problem:
\[ \frac{x}{-7} \geq -5 \]
Multiply both sides by $-7$:

$-7 \cdot \frac{x}{7} \leq (-7) \cdot (-5)$ (direction of the inequality is changed)

Simplify:

$x \leq 35$

6. Original problem:

$-\frac{x}{15} \leq 5$

Multiply both sides by $-15$:

$-15 \cdot \frac{x}{15} \geq -15 \cdot (5)$ (direction of inequality is changed)

Simplify:

$x \geq -75$, or $x \in [-75, \infty)$.

8. Original problem:

$\frac{x}{20} \geq -\frac{7}{40}$

Multiply both sides by $20$:

$20 \cdot \frac{x}{20} \geq 20 \cdot \left(-\frac{7}{40}\right)$ direction of the inequality is NOT changed

Simplify:

$x \geq -\frac{7}{2}$ or $\left[-\frac{7}{2}, \infty\right)$

10. Original problem:

$75x \geq 125$

Divide both sides by $75$:

$\frac{1}{75} \cdot (75x) \geq \frac{1}{75} \cdot (125)$ (direction of inequality is NOT changed)

Simplify:

$x \geq \frac{5}{3}$, or $\{x \text{ is a real number, } x \geq \frac{5}{3}\}$.

12. Original problem:

$\frac{x}{-15} < 8$

Multiply both sides by $-15$:

$-15 \cdot \frac{x}{-15} > (-15) \cdot 8$ direction of the inequality is changed

Simplify:

$x > -120$ or $\{x \text{ is a real number, } x > -120\}$
Multi-Step Inequalities

2. Original problem:

\[ 2x < 7x - 36 \]

Subtract 7x from both sides:

\[ 2x - 7x < 7x - 7x - 36 \]

Simplify:

\[ -5x < -36 \]

Divide both sides by \(-5\), remembering to switch the direction of the inequality:

\[ x > \frac{36}{5} \quad \text{or} \quad \{ x \text{ is a real number}, x > \frac{36}{5} \} \]

4. Original problem:

\[ 5 - x < 9 + x \]

Subtract x from both sides:

\[ 5 - x - x < 9 + x - x \]

\[ 5 - 2x < 9 \]

Subtract 5 from both sides:

\[ 5 - 5 - 2x < 9 - 5 \]

\[ -2x < 4 \]

Divide both sides by \(-2\), remembering to switch the direction of the inequality:

\[ x > -2 \quad \text{or} \quad \{ x \text{ is a real number}, x > -2 \} \]

6. Original problem:

\[ 5(4x + 3) \geq 9(x - 2) - x \]

Distribute:

\[ 20x + 15 \geq 9x - 18 - x \]

Combine like terms:

\[ 20x + 15 \geq 8x - 18 \]

Subtract 8x from both sides:


\[20x - 8x + 15 \geq 8x - 8x - 18\]

Simplify:

\[12x + 15 \geq -18\]

Subtract 15 from both sides:

\[12x + 15 - 15 \geq -18 - 15\]

Simplify:

\[12x \geq -33\]

Divide both sides by 12: (direction of the inequality is NOT changed)

\[x \geq \frac{-33}{12} = \frac{-11}{4}\]

or \[\{x \text{ is a real number, } x > \frac{-11}{4}\}\]

8. \(8x - 5(4x + 1) \geq -1 + 2(4x - 3)\)

Distribute:

\[8x - 20x - 5 \geq -1 + 8x - 6\]

Combine like terms:

\[-12x - 5 \geq 8x - 7\]

Subtract 8x from both sides:

\[-12x - 8x - 5 \geq 8x - 8x - 7\]

Simplify:

\[-20x - 5 \geq -7\]

Add 5 to both sides:

\[-20x - 5 + 5 \geq -7 + 5\]

Simplify:

\[-20x \geq -2\]

Divide both sides by \(-2\): (direction of the inequality is changed)

\[x \leq \frac{1}{10}\]

or \[\{x \text{ is a real number, } x \leq \frac{1}{10}\}\]
10. Original problem:

\[ \frac{2}{3}x - \frac{1}{2}(4x - 1) \geq x + 2(x - 3) \]

Multiply both sides by 6 (the LCD):

\[ 6 \cdot \frac{2}{3}x - 6 \cdot \frac{1}{2}(4x - 1) \geq 6x + 6 \cdot 2(x - 3) \]

Simplify:

\[ 4x - 3(4x - 1) \geq 6x + 12(x - 3) \]

Distribute:

\[ 4x - 12x + 3 \geq 6x + 12x - 36 \]

Combine like terms:

\[ -8x + 3 \geq 18x - 36 \]

Add 8x to both sides:

\[ -8x + 8x + 3 \geq 18x + 8x - 36 \]

Simplify:

\[ 3 \geq 26x - 36 \]

Add 36 to both sides:

\[ 3 + 36 \geq 26x - 36 + 36 \]

Simplify:

\[ 39 \geq 26x \]

Divide both sides by 26:

\[ \frac{39}{26} \geq x \quad \text{or} \quad \left\{ x \text{ is a real number, } x \leq \frac{3}{2} \right\} \]

12. Step 1: What we want to know: the score on the last exam (so that his average is at least 90)

Let \( x \) = the score on the last exam

Step 2: Formula for averaging is: sum of all the scores divided by the number of scores.

Since the average for the term must be at least 90, we have:

\[ \frac{82 + 95 + 86 + 88 + x}{5} \geq 90 \]
Step 3: We solve the inequality. Multiply both sides by 5 to clear the fraction:

\[
5 \cdot \frac{82 + 95 + 86 + 88 + x}{5} \geq 5 \cdot 90
\]

Simplify:

\[
82 + 95 + 86 + 88 + x \geq 450
\]

Add the scores and subtract the total from both sides:

\[
x \geq 450 - 351
\]

Simplify:

\[
x \geq 99
\]

Answer: Proteek must score at least 99 on the last exam.

Step 4: The solution checks out.

**Compound Inequalities**

2. \(x < -2\) or \(x \geq 5\)

4. \(x \leq -2\) or \(x > 1.5\)

6. Original problem:

\[
1 \leq 3x + 4 \leq 4
\]

Separate the two inequalities with an “and” and solve each separately:

\[
1 \leq 3x + 4 \quad \text{and} \quad 3x + 4 \leq 4
\]

\[
-3 \leq 3x \quad \text{and} \quad 3x \leq 0
\]

\[
-1 \leq x \quad \text{and} \quad x \leq 0
\]

Answer: \(-1 \leq x \) and \(x \leq 0\). This can be written as: \(-1 \leq x \leq 0\)

*(note SE has incorrect solution \(-\frac{4}{3} \leq x \leq -\frac{1}{3}\))*

8. Original problem:

\[
\frac{3}{4} \leq 2x + 9 \leq \frac{3}{2}
\]

Separate the two inequalities with an “and” and solve each separately:

\[
\frac{3}{4} \leq 2x + 9 \quad \text{and} \quad 2x + 9 \leq \frac{3}{2}
\]

\[
3 \leq 8x + 36 \quad \text{and} \quad 4x + 18 \leq 3
\]

\[
-33 \leq 8x \quad \text{and} \quad 4x \leq -15
\]
Answer: $-\frac{33}{8} \leq x$ and $x \leq -\frac{15}{4}$. This can be written as: $-\frac{33}{8} \leq x \leq -\frac{15}{4}$

10. Original problem:

$$4x - 1 \quad \text{or} \quad \frac{9x}{2} < 3$$

Solve each inequality separately:

$$4x - 1 \geq 7 \quad \text{or} \quad \frac{9x}{2} < 3$$

$$4x \geq 8 \quad \text{or} \quad 9x < 6$$

$$x \geq 2 \quad \text{or} \quad x < \frac{2}{3}$$

Answer: $x \geq 2$ or $x < \frac{2}{3}$.

12. (Problem statement is incomplete)

14. Original problem:

$$4x + 3 \leq 9 \quad \text{or} \quad -5x + 4 \leq -12$$

Separate the two inequalities with an “or” and solve each separately:

$$4x + 3 \leq 9 \quad \text{or} \quad -5x + 4 \leq -12$$

$$4x \leq 6 \quad \text{or} \quad -5x \leq -16$$

Answer: $x \leq \frac{3}{2}$ or $x \geq \frac{16}{5}$.

**Absolute Value Equations**

2. 12

4. 1/10

6. Distance is the absolute value of the difference between the two points:

$$\text{distance} = |5 - 22| = |-17| = 17$$

8. Distance is the absolute value of the difference between the two points:

$$\text{distance} = |-2 - 3| = |-5| = 5$$

10. There are two possibilities: the expression inside the absolute value signs is not negative or is negative. We solve each equation separately:

$$x + 2 = 6 \quad \text{or} \quad x + 2 = -6$$

Answer: $x = 4$ and $x = -8$.

The equation $|x + 2| = 6$ can be interpreted as, “What numbers on the number line are 6 units away from the number $-2$?” If we draw the number line, we see that there are two possibilities: $-8$ and $4$.
12. There are two possibilities: the expression inside the absolute value signs is not negative or is negative. We solve each equation separately:

\[
\begin{align*}
4x - 1 &= 19 \\
4x &= 20 \\
x &= 5
\end{align*}
\quad
\begin{align*}
4x - 1 &= -19 \\
4x &= -18 \\
x &= -\frac{9}{2}
\end{align*}
\]

Answer: \(x = 5\) and \(x = -\frac{9}{2}\)

The equation \(|4x - 1| = 19\) can be interpreted as, “What numbers on the number line are \(\frac{19}{4}\) units away from the number \(\frac{1}{4}\)?” If we draw the number line we see that there are two possibilities: \(x = 5\) and \(x = -\frac{9}{2}\).

14.

16.

**Absolute Value Inequalities**

2. \(|x| > 3.5\) represents all numbers whose distance from zero is greater than 3.5.
Answer: $x < -3.5$ or $x > 3.5$

4. Original problem:

$$\left| \frac{x}{5} \right| \leq 6$$

Rewrite as a compound inequality:

$$\frac{x}{5} \geq -6 \quad \text{and} \quad \frac{x}{5} \leq 6$$

Solve each inequality:

$$x \geq -30 \quad \text{and} \quad x \leq 30$$

Answer: $-30 \leq x \leq 30$

6. Original problem:

$$|x - 5| > 8$$

Rewrite as a compound inequality:

$$x - 5 > 8 \quad \text{or} \quad x - 5 < -8$$

Solve each inequality:

$$x > 13 \quad \text{or} \quad x < -3$$

8. Original problem:

$$\left| \frac{x - 3}{4} \right| \leq \frac{1}{2}$$

Rewrite as a compound inequality:

$$\frac{x - 3}{4} \geq -\frac{1}{2} \quad \text{and} \quad \frac{x - 3}{4} \leq \frac{1}{2}$$

Solve each inequality:

$$x \geq \frac{1}{4} \quad \text{and} \quad x \leq \frac{5}{4}$$

Answer: $\frac{1}{4} \leq x \leq \frac{5}{4}$

10. Original problem:

$$|5x + 3| < 7$$

Rewrite as a compound inequality:

$$5x + 3 < 7 \quad \text{and} \quad 5x + 3 > -7$$
Solve each inequality:

\[
5x < 4 \quad \text{or} \quad 5x > -10 \\
x < \frac{4}{5} \quad \text{or} \quad x > -2
\]

Solution: \(-2 < x < \frac{4}{5}\)

12. Original problem:

\[
\left| \frac{2x}{7} + 9 \right| > \frac{5}{7}
\]

Rewrite as a compound inequality:

\[
\frac{2x}{7} + 9 < -\frac{5}{7} \quad \text{or} \quad \frac{2x}{7} + 9 > \frac{5}{7}
\]

Solve each inequality:

\[
2x + 63 < -5 \quad \text{or} \quad 2x + 63 > 5 \\
2x < -68 \quad \text{or} \quad 2x > -58
\]

Answer: \(x < -34 \text{ or } x > -29\)

**Linear Inequalities in Two Variables**

2.

![Graph of linear inequality](image)

4.

![Graph of linear inequality](image)
6.

8.

10.
14. Apply the four-step problem-solving plan:

Step 1: Let $x =$ the number of daytime minutes used in a 24 hour period
Let $y =$ the number of nighttime minutes used in a 24 hour period

Step 2: The total charge for phone use during a 24 hour period is:
$50x + 10y$ (in cents)

We are looking for a total charge of more than $20. $20 is $20(100) = 2000$ cents. We write the inequality as:

$$50x + 10y > 2000$$

Step 3: To find the solution set, graph the inequality $50x + 10y > 2000$. Rewrite in slope-intercept form: $y > 200 - 5x$. Graph the line $y = 200 - 5x$ using a method of your choice. The line will be solid.
We shade above the line; since $y > 200 - 5x$.
Notice that only the first quadrant of the coordinate plane is used because minutes should be positive.
Also, note that any point in this region is a possible solution (in addition to those with integer coordinates).

Step 4: Check a few points from the region to be sure that more than $20 is charged.
Chapter 7

TE Solving Systems of Equations and Inequalities - Solution Key

7.1 Complete Solutions to Even-Numbered Review Questions

Linear Systems by Graphing

2a. To check if the ordered pair $(8, 13)$ is a solution to the given system of equations, plug each of the $x$ and $y$ values into the equations to see if they make both equations true.

\[
\begin{align*}
y &= 2x - 3 \\
? &= 13 = 2(8) - 3 \\
13 &= 13 \text{(true equation)} \\
\end{align*}
\]

\[
\begin{align*}
y &= x + 5 \\
? &= 13 = 8 + 5 \\
13 &= 13 \text{(true equation)}
\end{align*}
\]

The ordered pair $(8, 13)$ is a solution to the system since it satisfies both equations (the point lies on both lines).

2b. To check if the ordered pair $(-7, 6)$ is a solution to the given system of equations, plug each of the $x$ and $y$ values into the equations to see if they make both equations true.

\[
\begin{align*}
y &= 2x - 3 \\
? &= 6 = 2(-7) - 3 \\
6 &= -17 \text{(false equation)}
\end{align*}
\]
The ordered pair \((-7, 6)\) is not a solution to the system since it does not satisfy the first equation. Note that it is not necessary to check the second equation because the ordered pair must make both equations true (the point needs to be on both lines) for it to be a solution to the system.

2c. To check if the ordered pair \((0, 4)\) is a solution to the given system of equations, plug each of the \(x\) and \(y\) values into the equations to see if they make both equations true.

\[
y = 2x - 3 \\
4 = 2(0) - 3 \\
4 = -3 \text{(false equation)}
\]

The ordered pair \((0, 4)\) is not a solution to the system since it does not satisfy the first equation. Note that it is not necessary to check the second equation because the ordered pair must make both equations true (the point needs to be on both lines) for it to be a solution to the system.

4a. Check ordered pair \((3, -\frac{3}{2})\), by substituting each of the \(x\) and \(y\) values into the equations to see if they make both equations true.

\[
3x + 2y = 6 \\
3(3) + 2\left(-\frac{3}{2}\right) = 6 \\
6 = 6 \text{(true equation)}
\]

\[
y = \frac{x}{2} - 3 \\
\frac{3}{2} = \frac{3}{2} - 3 \\
-3 = -3 \text{(true equation)}
\]

The ordered pair \((3, -\frac{3}{2})\) is a solution to the system since it satisfies both equations (the point lies on both lines).

4b. To check if the ordered pair \((-4, 3)\) is a solution to the given system of equations, plug each of the \(x\) and \(y\) values into the equations to see if they make both equations true.

\[
3x + 2y = 6 \\
3(-4) + 2(3) = 6 \\
-6 \neq 6 \text{(false equation)}
\]

The ordered pair \((-4, 3)\) is not a solution to the system since it does not satisfy the first equation.
6. The intersection is (3, 3)

8. The intersection is (2, 3)

10. The intersection is (2, 3)
12. The intersection is (4, 4).

14. The intersection is (−4, 4)

16. Step 1: What we know: First company charges $120 and $30/month. The second company charges $40 and $45/month.

What we need: the number of months elapsed when the total cost of both plans is the same.

Step 2: Label variables.

Let \( x \) = number of months from now. Let \( y \) = total cost for \( x \) months of use.

First plan: \( y = 30x + 120 \)
Second plan: \( y = 45x + 40 \)

Step 3: Solve.

Method of solution: graphing by hand or using a graphing calculator.

\[
x = \frac{16}{3} \approx 5.33
\]

Answer: The cost of both plans will be the same in about 5.33 months.

Step 4: Check. The solution checks out.

**Solving Linear Systems by Substitution**

2. Start by looking to isolate one variable in either equation. In the first equation, the coefficient of \( x \) is 1. It makes sense to use this equation to solve for \( x \):
\[ x - 3y = 10 \]

Add 3y to both sides:

\[ x = 3y + 10 \]

Substitute this expression into the second equation:

\[ 2x + y = 13 \]
\[ 2(3y + 10) + y = 13 \]

Distribute the 2:

\[ 6y + 20 + y = 13 \]

Collect like terms:

\[ 7y + 20 = 13 \]

Subtract 20 from both sides:

\[ 7y = -7 \]

Divide by 7:

\[ y = -1 \]

Substitute back into our expression for x and simplify:

\[ x = 3y + 10 \]
\[ x = 3(-1) + 10 \]
\[ x = -3 + 10 \]
\[ x = 7 \]

Answer: \((7, -1)\) is the solution to the system.

4. Step 1: What we know: the sum of two numbers is 70. Their difference is 11.
What we want to know: the two numbers.
Step 2: Label variables.
Let \(x\) and \(y\) be the two numbers. Then:
\[ x + y = 70 \] and 
\[ x - y = 11 \] (Notice how it is implicitly assumed that \(x > y\) in this equation.)
Step 3: Solve.
Method of solution: substitution. Start by looking to isolate one variable in either equation. In the first equation, the coefficient of either variable is 1. Use this equation to solve for \(x\):
\[ x + y = 70 \]

Add \( y \) to both sides:

\[ x = 70 - y \]

Substitute this expression into the second equation:

\[
\begin{align*}
x - y &= 11 \\
(70 - y) - y &= 11
\end{align*}
\]

Collect like terms:

\[ 70 - 2y = 11 \]

Subtract 70 from both sides:

\[ -2y = -59 \]

Divide by \(-2\):

\[ y = \frac{59}{2} = 29.5 \]

Substitute back into our expression for \( x \) and simplify:

\[
\begin{align*}
x &= 70 - y \\
x &= 70 - 29.5 \\
x &= 40.5
\end{align*}
\]

Answer: \((40.5, 29.5)\) is the solution to the system; but these are also the two numbers sought.

Step 4: Check. The solution checks out.

6. Step 1: What we know: the difference of the angles \( x \) and \( y \) is 18 degrees. The sum of the angles is 180 degrees.

What we want to know: the measures of \( x \) and \( y \).

Step 2: Label variables and write equations:

Let \( x \) and \( y \) be the angles shown in the figure. Then:

\[ x + y = 180 \]

\[ y - x = 18 \] (Notice that \( x < y \) in the figure.)

Step 3: Solve.

Method of solution: substitution. Start by looking to isolate one variable in either equation. In the first equation, the coefficient of either variable is 1. Use this equation to solve for \( x \):

\[ x + y = 180 \]
Add \( y \) to both sides:

\[ x = 180 - y \]

Substitute this expression into the second equation:

\[
\begin{align*}
  y - x &= 18 \\
  y - (180 - y) &= 18
\end{align*}
\]

Distribute the minus sign:

\[ y - 180 + y = 18 \]

Collect like terms:

\[ 2y - 180 = 18 \]

Add 180 to both sides:

\[ 2y = 198 \]

Divide by 2:

\[ y = 99 \]

Substitute back into our expression for \( x \) and simplify:

\[
\begin{align*}
  x &= 180 - y \\
  x &= 180 - 99 \\
  x &= 81
\end{align*}
\]

Answer: \((81, 99)\) is the solution to the system; but these are also the measures of the two angles sought.

Step 4: Check. The solution checks out.

8. Step 1: What we know: acid comes in concentrations of 10% and 35%. The experiment calls for one liter of 15% acid.

What we want to know: the number of liters of each concentration needing to be mixed resulting in one liter of 15% acid.

Step 2: To set this problem up, we first need to define our variables. Our unknowns are the amount of the 10% concentrated acid \((x)\) and the amount of the 35% concentrated acid \((y)\).

Convert the percentages (10%, 35% and 15%) into decimals (0.1, 0.35 and 0.15).

The two pieces of critical information are the final volume (1 liter) and the final amount of 15% concentrated acid (15% of 1 \( l = 0.15 \)). The equations will look like this:

Volume equation: \( x + y = 1 \)

Concentration equation: \( 0.1x + 0.35y - 0.15(1) \)

Step 3: Solve.
Method of solution: substitution. Start by looking to isolate one variable in either equation. In the first equation, the coefficient of either variable is 1. Use this equation to solve for $x$:

$$x + y = 1$$

Add $y$ to both sides:

$$x = 1 - y$$

Substitute this expression into the second equation:

$$0.1x + 0.35y = 0.15(1)$$
$$0.1(1 - y) + 0.35y = 0.15$$

Distribute the minus sign:

$$0.1 - 0.1y + 0.35y = 0.15$$

Collect like terms:

$$0.1 + 0.25y = 0.15$$

Subtract 0.1 from both sides:

$$0.25y = 0.05$$

Divide by 0.25:

$$y = \frac{1}{5} = 0.2$$

Substitute back into our expression for $x$ and simplify:

$$x = 1 - y$$
$$x = 1 - 0.2$$
$$x = 0.8$$

Answer: $(0.8, 0.2)$ is the solution to the system; but these also represent the concentrations of the acids: 0.8 liters of 10% acid and 0.2 liters of 35% acid are needed to mix in order to produce 1 liter of an acid with 15% concentration.

Step 4: Check. The solution checks out.

**Solving Linear Systems by Elimination through Addition or Subtraction**

2. Subtract the second equation from the first: everything on the left of the equal sign in the first equation minus everything on the left of the equal sign in the second equation will equal the difference of the numbers
on the right of both equal signs. The simple way to perform this calculation is by adding in columns – remembering to keep like variables in their own columns.

\[
\begin{align*}
5x + 7y &= -31 \\
- (5x - 9y &= 17) \\
0x + 16y &= -48
\end{align*}
\]

So \(y = -3\). To find a value for \(x\) substitute the value for \(y\) back into either equation:

\[
\begin{align*}
5x + 7y &= -31 \\
5x + 7(-3) &= -31 \\
5x - 21 &= -31 \\
5x &= -10 \\
x &= -2
\end{align*}
\]

Answer: \((-2, -3)\) is the solution to the system.

4. Step 1: What we know: three candy bars and four fruit roll-ups cost $2.84. Three candy bars and 1 fruit roll-up cost $1.79.
What we want to know: the cost of a candy bar and the cost of a fruit roll-up.
Step 2: Label variables and set-up equations:
Let \(c\) = cost of one candy bar. Let \(f\) = cost of one fruit roll-up. Then:

\[
\begin{align*}
3c + 4f &= 2.84 \\
3c + f &= 1.79
\end{align*}
\]

Step 3: Solve.
Method of solution: Elimination. Subtract the second equation from the first.

\[
\begin{align*}
3c + 4f &= 2.84 \\
- (3c + f &= 1.79) \\
0c + 3f &= 1.05
\end{align*}
\]

So \(f = 0.35\). To find a value for \(c\) substitute the value for \(f\) back into either equation:

\[
\begin{align*}
3c + f &= 1.76 \\
3c + 0.35 &= 1.79 \\
3c &= 1.44 \\
c &= 0.48
\end{align*}
\]

Answer: The candy bar costs $0.48 and the fruit roll-up costs $0.35.
Step 4: Check. The solution checks out.

6. Step 1: What we know: a taxi will charge a pick-up fee and a per-mile fee. A 12 mile trip costs $14.29 and a 17 mile trip costs $19.91.
What we want to know: the amount of each fee and the cost of a 7 mile trip.

Step 2: Label variables and set-up equations:
Let $x =$ the amount of the pick-up fee. Let $y =$ the amount of the per-mile fee. Then:

\[ x + 12y = 14.29 \]
\[ x + 17y = 19.91 \]

Step 3: Solve.
Method of solution: Elimination. Subtract the second equation from the first.

\[ x + 12y = 14.29 \]
\[ -(x + 17y = 19.91) \]
\[ 0c - 5y = -5.62 \]

So $y = 1.124$ To find a value for $x$ substitute the value for $y$ back into either equation:

\[ x + 12(1.124) = 14.29 \]
\[ x + 13.488 = 14.29 \]
\[ x = 0.802 \]

Answer: The taxi charges $0.802 to pick up someone. The taxi charges $1.124 per mile. To find out how much a 7 mile trip would cost, compute: $0.802 + 1.124(7) = $8.67.

Step 4: Check. The solution checks out.

8. Step 1: What we know: the plumber earns $35 per hour. The builder earns $28 per hour. Together they were paid $330.75. The plumber earned $106.75 more than the builder.
What we want to know: the number of hours worked by the plumber and the builder.

Step 2: Label variables and set-up equations.
Let $p =$ the number of hours worked by the plumber. Let $b =$ the number of hours worked by the builder.

\[ 35p + 28b = 330.75 \]
\[ 35p - 28b = 106.75 \]


\[ 35p + 28b = 330.75 \]
\[ -(35p - 28b = 106.75) \]
\[ 0p + 56b = 224 \]

So $b = 4$. To find a value for $x$ substitute the value for $y$ back into either equation:

\[ 35p + 28(4) = 330.75 \]
\[ 35p + 112 = 330.75 \]
\[ 35p = 218.75 \]
\[ p = 6.25 \]

Answer: The plumber worked 6 hours and 15 minutes. The builder worked 4 hours.
Solving Systems of Equations by Multiplication

2a. The system is set up ideally for the substitution method. Substitute the first equation for \( x \) in the second equation.

\[(3y) - 2y = -3\]

Collect like terms:

\[y = -3\]

To find \( x \), substitute the value of \( y \) in either equation:

\[x = 3y\]
\[x = 3(-3)\]
\[x = -9\]

Answer: the solution to the system is \((-9, -3)\).

2b. The system is set-up ideally for the substitution method. Substitute the first equation for \( y \) in the second equation.

\[(3x + 2) = -2x + 7\]

Collect like terms:

\[5x = 5\]
\[x = 1\]

To find \( y \), substitute the value of \( x \) in either equation:

\[y = 3x + 2\]
\[y = 3(1) + 2\]
\[y = 5\]

Answer: the solution to the system is \((1, 5)\).

2c. The system is set-up ideally for the elimination method. Subtract the second equation from the first equation.

\[5x - 5y = 5\]
\[-(5x + 5y = 35)\]
\[0x - 10y = -30\]

So, \( y = 3 \). To find \( x \), substitute the value of \( y \) in either equation:

\[5x - 5y = 5\]
\[5x - 5(3) = 5\]
\[5x - 15 = 5\]
\[5x = 20\]
\[x = 4\]
Answer: the solution to the system is (4, 3).

2d. The system is set-up ideally for the elimination method. Writing the second equation as $0 = 3x - 2y + 12$, we add the two equations:

\[
\begin{align*}
y &= -3x - 3 \\
+ (0 &= 3x - 2y + 12) \\
\hline
y &= -2y + 9 \\
3y &= 9 \\
y &= 3
\end{align*}
\]

Substitute back into the first equation:

\[
\begin{align*}
(3) &= -3x - 3 \\
-3x &= 6 \\
x &= -2
\end{align*}
\]

Answer: the solution to the system is (-2, 3).

2e. The system is set-up ideally for the elimination method. Add the two equations:

\[
\begin{align*}
3x - 4y &= 3 \\
+ (5x + 4y &= 10) \\
\hline
8x &= 13 \\
x &= 13/8
\end{align*}
\]

Substitute back into the first equation:

\[
\begin{align*}
3(13/8) - 4y &= 3 \\
39/8 - 4y &= 3 \\
-4y &= 24/8 - 39/8 = -15/8 \\
y &= 15/32
\end{align*}
\]

Answer: the solution to the system is (13/8, 15/32).

2f. Solution: elimination (using multiplication). By multiplying the first equation by $-3$, the coefficient of $y$ will be opposite of the coefficient of $y$ in the second equation.

\[
\begin{align*}
9x - 2y &= -4 \\
2x - 6y &= 1 \\
-27x + 6y &= 12 \\
2x - 6y &= 1
\end{align*}
\]

Adding the two equations results in:

\[
\begin{align*}
-25x &= 13 \\
x &= -\frac{13}{25}
\end{align*}
\]
To find $y$, substitute the value of $x$ in either equation:

\[
2x - 6y = 1 \\
2\left(-\frac{13}{25}\right) - 6y = 1 \\
-\frac{26}{25} - 6y = 1 \\
-6y = 1 + \frac{26}{25} \\
-6y = \frac{25}{25} + \frac{26}{25} \\
-6y = \frac{51}{25} \\
y = -\frac{17}{50}
\]

Answer: the solution to the system is \(\left(-\frac{13}{25}, -\frac{17}{50}\right)\).

4. Step 1: What we know: fertilizer comes in two concentrations, 5% and 15%. The mix will consist of 100 liters of a fertilizer with a 12% concentration.

What we need: the amount of 5% and the amount of 15% fertilizer.

Step 2: Label variables and set-up equations. Convert percents to decimals.

Let $x$ = the amount of 5% fertilizer. Let $y$ = the amount of 15% fertilizer. Then:

\[
x + y = 100 \\
0.05x + 0.15y = 0.12(100)
\]


\[
y = 100 - x \\
0.05x + 0.15(100 - x) = 12
\]

Distribute, collect like terms and simplify:

\[
0.05x + 15 - 0.15x = 12 \\
-0.10x = -3 \\
x = 30
\]

To solve for $y$, substitute the value of $x$ in either equation.

\[
x + y = 100 \\
30 + y = 100 \\
y = 70
\]

Answer: the solution to the system is (30, 70) which also represents the amount of 5% and the amount of 15% fertilizer, respectively.

Step 4. Check. The solution checks out.
6. Step 1: What we know: A total of $100,000 was invested in two companies. Company A showed a gain of 13% while Company B showed a loss of 3%. The overall gain was +8%.

What we need: the amount of money invested in each company.

Step 2: Label variables and set-up equations. Convert percents to decimals. A gain of 13% is equivalent to multiplying the initial amount by 1.13; similarly, a loss of 3% is equivalent to multiplying the initial amount by 0.97.

Let $x =$ the amount invested in company A, and
$y =$ the amount invested in company B

\[ x + y = 100000 \]
\[ 1.13x + 0.97y = 1.08(100000) \]


\[ y = 100000 - x \]
\[ 1.13x + 0.97(100000 - x) = 108000 \]

Distribute, collect like terms and simplify:

\[ 1.13x + 97000 - 0.97x = 108000 \]
\[ 0.16x = 11000 \]
\[ x = 68750 \]

Substitute into the first equation:

\[ y = 100000 - 68750 \]
\[ y = 31250 \]

Answer: the solution to the system is (68750, 31250) which means that Mr. Stein invested $68,750 in Company A and $31,250 in Company B.

8. Step 1: What we know: twice John’s age plus 5 times Claire’s age is 204. Nine times John’s age minus 3 times Claire’s age is also 204.

What we want: the ages of John and Claire.

Step 2: Label variables and set-up equations.

Let $x =$ John’s age now. Let $y =$ Claire’s age now. Then the first sentence translates as:

\[ 2x + 5y = 204 \]

The second sentence translates as:

\[ 9x - 3y = 204 \]

Putting the equations together yields a system of equations:

\[ 2x + 5y = 204 \]
\[ 9x - 3y = 204 \]
Step 3: Solve by substitution: the second equation can be reduced by 3; divide both sides by 3.

\[2x + 5y = 204\]
\[3x - y = 68\]

Solve the second equation for \(y\):

\[2x + 5y = 204\]
\[y = 3x - 68\]

Substitute the expression for \(y\) into the first equation:

\[2x + 5(3x - 68) = 204\]

Distribute, collect like terms and simplify:

\[2x + 15x - 340 = 204\]
\[17x = 544\]
\[x = 32\]

To find \(y\), substitute the value of \(x\) in either equation:

\[2(32) + 5y = 204\]
\[64 + 5y = 204\]
\[5y = 140\]
\[y = 28\]

Answer: the solution to the system is \((32, 28)\) which also represents the ages of John and Claire, respectively.

Step 4: Check. The solutions checks out.

**Special Types of Linear Systems**

2a. Method of solution: elimination (Notice that none of the coefficients are 1; which would make the method of substitution the preferred choice of solution.)

\[3x + 2y = 4\]
\[-2x + 2y = 24\]

Subtract the second equation from the first:

\[3x + 2y = 4\]
\[-(-2x + 2y = 24)\]

\[5x + 0y = -20\]

So \(x = -4\). To find \(y\), substitute the value for \(x\) in either equation:
Answer: The system of equations has one solution, \((-4, 8)\), and therefore is a consistent system.

2b. Method of solution: elimination

Multiply the first equation by 2: \(10x - 4y = 6\)

Multiply the second equation by 5: \(10x - 15y = 50\)

Subtract the second equation from the first:

\[
\begin{align*}
10x - 4y &= 6 \\
- (10x - 15y &= 50) \\
11y &= -44 \\
y &= -4
\end{align*}
\]

Substitute into the first equation:

\[
\begin{align*}
5x - 2(-4) &= 3 \\
5x + 8 &= 3 \\
5x &= -5 \\
x &= -1
\end{align*}
\]

Answer: The system of equations has one solution, \((-1, -4)\), and therefore is a consistent system.

2c. Method of solution: substitution. Notice that the coefficient of \(y\) in the second equation is 1.

\[
\begin{align*}
3x - 4y &= 13 \\
y &= -3x - 7
\end{align*}
\]

Substitute the expression for \(y\) into the first equation:

\[
3x - 4(-3x - 7) = 13
\]

Distribute, collect like terms, and simplify:

\[
\begin{align*}
3x + 12x + 28 &= 13 \\
15x &= -15 \\
x &= -1
\end{align*}
\]

To find \(y\), substitute the value for \(x\) into either equation:

\[
\frac{3x}{2} + 2y = 4
\]

\[
\begin{align*}
3(-4) + 2y &= 4 \\
-12 + 2y &= 4 \\
2y &= 16 \\
y &= 8
\end{align*}
\]
\[
\begin{align*}
\text{Answer: The system of equations has one solution, } (-1, -4), \text{ and therefore is a consistent system.}
\end{align*}
\]

2d. Method of solution: elimination

Multiply the first equation by \(-2\): \(-10x + 8y = -2\)

Add the first equation to the second:

\[
\begin{align*}
-10x + 8y &= -2 \\
+ (-10x + 8y &= 30) \\
\hline
0 &= 28
\end{align*}
\]

This is a false equation, so the system is inconsistent.

2e. Method of solution: elimination.

\[
\begin{align*}
4x + 5y &= 0 \\
3x &= 6y + 4.5
\end{align*}
\]

Align the variables column-wise:

\[
\begin{align*}
4x + 5y &= 0 \\
3x - 6y &= 4.5
\end{align*}
\]

To eliminate \(x\), find the LCM of 3 and 4, which is 12. This means the first equation should be multiplied by 3 and the second by \(-4\) (to ensure the coefficients of \(x\) are opposites so they will cancel).

\[
\begin{align*}
3(4x + 5y &= 0) \\
-4(3x - 6y &= 4.5)
\end{align*}
\]

Distribute:

\[
\begin{align*}
12x + 15y &= 0 \\
-12x + 24y &= -18
\end{align*}
\]

Adding the equations results in:

\[
\begin{align*}
0x + 39y &= -18 \\
y &= -\frac{18}{39} = -\frac{6}{13}
\end{align*}
\]

To find \(x\), substitute the value of \(y\) into either equation:

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\[3x = 6y + 4.5\]
\[3x = 6\left(-\frac{6}{13}\right) + 4.5\]
\[3x = \left(-\frac{36}{13}\right) + \frac{(4.5)(13)}{13}\]
\[3x = \frac{22.5}{13}\]
\[\frac{1}{3} \cdot 3x = \frac{1}{3} \left(-\frac{22.5}{13}\right)\]

\[x = \frac{15}{26}\]

Answer: The system of equations has one solution, \(\left(\frac{15}{26}, -\frac{6}{13}\right)\), and therefore is a consistent system.

2f. Method of solution: substitution
Solve for \(y\) in the second equation: \(y = 2x - 4\)
Substitute into the first equation:

\[-2(2x - 4) + 4x = 8\]
\[-4x + 8 + 4x = 8\]
\[0 = 0\]

This is a true equation, so the system has infinitely many solutions and is dependent.

2g. Method of solution: substitution
To simplify the algebra, multiply the first equation by 2: \(2x - y = 3\)
Solve for \(y\) in the second equation: \(y = -3x + 6\)
Substitute into the first equation:

\[2x - (-3x + 6) = 3\]
\[2x + 3x - 6 = 3\]
\[5x = 9\]
\[x = \frac{9}{5}\]

Plug back into the second equation:

\[y = -3\left(\frac{9}{5}\right) + 6\]
\[y = -\frac{27}{5} + \frac{30}{5}\]
\[y = \frac{3}{5}\]

Answer: The system of equations has one solution, \((\frac{9}{5}, \frac{3}{5})\), and therefore is a consistent system.

2h. Method of solution: elimination
Multiply the first equation by 20: \(x + 5y = 120\)
Subtract the second equation from the first:
\[ x - 5y = 120 \\
- (x + y = 24) \]

\[ 4y = 96 \]
\[ y = 24 \]

Substitute into the second equation:

\[ x + (24) = 24 \]
\[ x = 0 \]

Answer: The system of equations has one solution \((0, 24)\), and is therefore a consistent system.

2i. Method of solution: elimination

Multiply the first equation by 3: \(3x + 2y = 18\)

Subtract the second equation from the first:

\[ 3x + 2y = 18 \]
\[ -(3x + 2y = 2) \]

\[ 0 = 16 \]

This is a false equation, so the solution is inconsistent (note SE has incorrect answer “dependent” here)

16. Step 1: What do we know: 1200 people saw movies and paid a total of $8375. Children’s tickets cost $4.50 and adult tickets are $8.00

What we need: the number of children and the number of adults who saw movies.

Step 2: Label variables and set-up equations:

Let \(x\) = the number of children. Let \(y\) = the number of adults.

\[ x + y = 1200 \text{ (the total number of people was 1200)} \]
\[ 4.5x + 8y = 8375 \text{ (the total amount paid was$8375)} \]

Step 3: Method of solution: substitution

Solve for \(y\) in the first equation: \(y = 1200 - x\)

Substitute into the second equation:

\[ 4.5x + 8(1200 - x) = 8375 \]

Distribute and collect like terms:

\[ 4.5x + 9600 - 8x = 8375 \]
\[ -3.5x = -1225 \]
\[ x = 350 \]

To find \(y\), substitute the value for \(x\) in the first equation:
Answer: 350 children and 850 adults saw movies.

Step 4: Check. The solution checks out.

18. Step 1: What do we know: an airplane flew 2400 miles in 4 hours with the jet-stream. The same trip back against the jet-stream took 5 hours.

What we need: the airplane’s speed in still air. The jet-stream’s speed.

Step 2: Label variables and set-up equations:
Let $x =$ the speed of the plane in still air. Let $y =$ the jet-stream’s speed. We know that:

$\text{distance} = (\text{speed})(\text{time})$

So,

(with jet-stream equation) $2400 = (x + y)(4)$
(against jet-stream equation) $2400 = (x - y)(4)$

Step 3: Method of solution: elimination.

$2400 = (x + y)(4)$
$2400 = (x - y)(4)$

Divide both sides by the time:

$x + y = 600$
$x - y = 480$

Add the equations:

$2x + 0y = 1080$
$x = 540$

To find $y$, substitute the value for $x$ in either equation:

$2400 = (540 + y)(4)$

Solve for $y$:

$t = 60$

Answer: the speed of the plane in still air is 540 mph and the speed of the jet-stream is 60 mph.

Step 4: Check. The solution checks out.
Systems of Linear Inequalities

2.

\[ -15x + 16 - 8y \]

4.

\[ 5x + 2y = -25 \]

\[ 3x - 2y = 17 \]

\[ x - 6y = 27 \]
6.

8. Step 1: What we know: At most 18 pieces of furniture can be made. Materials cost $20/bookcase and $45/cabinet. Andrew has no more than $600 to spend on materials. Profit is $60/bookcase and $100/cabinet.

What we need: how many of each piece of furniture should be made to maximize profit?

Step 2: Label variables and set-up equations.

Let \( x \) = number of bookcases to be made. Let \( y \) = number of cabinets to be made. Then we want to:

Maximize:

\[
\text{Profit} = 60x + 100y
\]

Subject to the following constraints:

\[
x + y \leq 18
\]
\[
20x + 45y \leq 606
\]
\[
x \geq 0, \text{ integer}
\]
\[
y \geq 0, \text{ integer}
\]

Step 3: Solve. Start by finding the solution region to the set of inequalities by graphing each line and shading appropriately. Your shaded region is the feasibility region where all the possible solutions can occur.

\[
y \leq -x + 18
\]
\[
y \leq -\frac{20}{45}x + \frac{600}{45}
\]
\[
x \geq 0
\]
\[
y \geq 0
\]

As the number of bookcases and cabinets that Andrew should make must be an integer, the solution is not necessarily on the boundary. We need to check several lattice points from the interior of the feasible region in addition to those lattice points on the boundary. The problem-solving strategies make a table, make a graph will come in handy here.
Since the profit equation is “all positive” (positive coefficients and terms are added together), once we test a point \((x, y)\), we will not need to test any of the points “below” it or to the “left” of it; meaning that none of the points \((x, k)\) for all \(k < y\) need to be checked and none of the points \((k, y)\) for all \(k < x\) need to be checked. Looking carefully at the graph of the feasible region, we are essentially looking for lattice points near the boundary lines as far from the axes as possible (informally speaking). Of course, the points we pick from the feasible region will satisfy the constraints, although it is suggested that students double-check that the points they pick are indeed from inside the region.

Table 7.1:

<table>
<thead>
<tr>
<th>Points ((x, y))</th>
<th>Profit: (60x + 100y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 13))</td>
<td>1300</td>
</tr>
<tr>
<td>((3, 12))</td>
<td>1380</td>
</tr>
<tr>
<td>((5, 11))</td>
<td>1400</td>
</tr>
<tr>
<td>((7, 10))</td>
<td>1420</td>
</tr>
<tr>
<td>((9, 9))</td>
<td>1440</td>
</tr>
</tbody>
</table>

Notice that for \(x > 8\), only the point \((9, 9)\) needs to be checked. The points \((10, 8), (11, 7), (12, 6), \ldots, (18, 0)\) that lie on the line \(y = -x + 18\) do not need to be checked even though they are clearly in the feasible region. This is because the variable \(y\) takes on smaller values as the variable \(x\) increases in value; meaning you would be making fewer bookcases, which bring in more profit, than cabinets, which bring in less profit. The profit from such combinations is less than that of 9 bookcases and 9 cabinets.

Answer: Andrew should make 9 bookcases and 9 cabinets (for a total profit of $1440).
Chapter 8

TE Exponential Functions - Solution Key

8.1 Complete Solutions to Even-Numbered Review Questions

Exponent Properties Involving Products

2. \(3x \cdot 3x \cdot 3x = (3x)^3\), since there are three factors of 3x.
4. \(6 \cdot 6 \cdot 6 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y = 6^3 \cdot x^2 \cdot y^4\) because there are 3 factors of 6, 2 factors of x, and 4 factors of y.
Answer: \(216x^2y^4\)
6. \((-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64\)
8. \((-0.6)^3 = (-06)(-06)(-06) = -0.216\).
10. Apply the product rule: \(2^2 \cdot 2^4 \cdot 2^6 = 2^{2+4+6} = 2^{12} = 4096\).
12. Apply the product rule: \(x^2 \cdot x^4 = x^{2+4} = x^6\).
14. Start by grouping like terms together:
\[
(4a^2)(-3a)(-5a^4) = (4)(-3)(-5) \cdot (a^2 \cdot a \cdot a^4)
\]
Multiply the numbers and apply the product rule on the group:
Answer: \(60a^7\)
16. Apply the power rule: \((xy)^2 = x^2y^2\).
18. Apply the power rule:
\[
(-2xy^4z^2)^5 = (-2)^5 \cdot x^5 \cdot y^{20} \cdot z^{10} = -32x^5y^{20}z^{10}
\]
20. Apply the power rule first:
\[
(4a^2)(-2a^3)^4 = (4a^2) \cdot (-2)^4 \cdot a^{12}
\]
Group like factors and apply the product rule to combine the groups:
\[ = (4)(-2)^4 \cdot (a^2 \cdot a^{12}) = 64a^{14} \]

22. Apply the power rule first on the middle parentheses:

\[
(2xy^2)(-x^2y)^2(3x^2y^2) = (2xy^2) \cdot (-1)^2 \cdot x^4 \cdot y^2 \cdot (3x^2y^2)
\]

Group like factors and apply the product rule to combine the groups:

\[ = 2 \cdot (-1)^2 \cdot 3 \cdot (x \cdot x^4 \cdot x^2)(y^2 \cdot y^2 \cdot y^2) = 6x^7y^6 \]

**Exponent Properties Involving Quotients**

2. Apply the quotient rule: \( \frac{6^7}{6^3} = 6^{7-3} = 6^4 = 1296 \).

4. Use the power rule for quotients first; then evaluate the result.

\[
\left( \frac{2^3}{3^3} \right)^3 = \frac{2^6}{3^9} = \frac{64}{19683}
\]

6. Apply the quotient rule: \( \frac{x^5}{x^2} = \frac{1}{x^{2-x}} = \frac{1}{x^r} \).

8. Group like terms together:

\[
\frac{x^6y^2}{x^2y^5} = \frac{x^6 \cdot y^2}{x^2 \cdot y^5}
\]

Reduce the numbers and apply the quotient rule on each grouping:

\[ = x^4 \cdot \frac{1}{y^3} = \frac{x^4}{y^3} \]

10. Group like terms together: \( \frac{15x^5}{6x} = \frac{15}{6} \cdot \frac{x^5}{x} \)

Simplify each term: \( \frac{15}{6} = 3, \frac{x^5}{x} = x^{5-1} = x^4 \). The product is \( 3x^4 \).

12. Group like terms together: \( \frac{25x^6}{20y^6x^2} = \frac{25}{20} \cdot \frac{x^6}{y^6 \cdot x^2} \)

Reduce the numbers and apply the quotient rule on each grouping:

\[ = \frac{5}{4} \cdot \frac{1}{y^3} \cdot x^4 = \frac{5x^4}{4y^4} \]

14. Apply the power rule for quotients first:

\[
\left( \frac{6a^2}{4b^4} \right)^2 \cdot \frac{5b}{3a} = \frac{6^2a^4}{4^2b^8} \cdot \frac{5b}{3a}
\]

Group like terms together:

\[ = \frac{6^2 \cdot 5}{4^2 \cdot 3} \cdot \frac{a^4}{a} \cdot \frac{b}{b^8} \]

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Reduce the numbers and apply the quotient rule on each grouping:

\[
\frac{6^2 \cdot 5}{4^2 \cdot 3} \cdot \frac{a^4}{a} \cdot \frac{b}{b^8} = \frac{15}{4} \cdot \frac{1}{b^7} = \frac{15a^3}{4b^7}
\]

16. Apply the product rule for quotients first:

\[
\frac{(2a^2bc^2)(6abc^3)}{4ab^2c} = \frac{12a^3b^2c^5}{4ab^2c}
\]

Reduce the numbers and apply the quotient rule on each grouping:

\[
= 3a^2c^4
\]

**Zero, Negative, and Fractional Exponents**

2. By definition, \(x^{-4} = \frac{1}{x^4}\).

4. Apply the negative rule for exponents on all the terms that have negative exponents:

\[
\frac{x^{-3}y^{-5}}{z^{-7}} = \frac{z^7}{x^3y^5}
\]

6. Use the negative power rule for fractions:

\[
\left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2 = \frac{b^2}{a^2}
\]

8. Apply the product rule for exponents:

\[
x^{-3} \cdot x^3 = x^{-3+3} = x^0 = 1
\]

10. Group like terms together:

\[
\frac{5x^6y^2}{x^6} = \frac{5x^6y^2}{x^6} = 5x^0y^2
\]

Apply the quotient rule on each grouping:

\[
= 5x^{6-8}y^{2-1} = 5x^{-2}y.
\]

12. Apply the power rule:

\[
\left(\frac{3x}{y^\frac{1}{3}}\right)^3 = \frac{3^3x^3}{y^{\frac{1}{3}}^3} = \frac{27x^3}{y}
\]

Apply the negative rule for exponents on the term in the denominator:

\[
= \frac{27x^3}{y} = 27x^3y^{-1}
\]

14. Apply the product rule in the numerator and the power rule in the denominator:

\[
\frac{(3x^3)(4x^4)}{(2y)^2} = \frac{12x^7}{4y^2}
\]
Then apply the quotient rule and negative rule for exponents:

\[
\frac{12x^7}{4y^2} = 3x^7y^{-2}
\]

16. Apply the quotient rule for exponents bringing the smaller exponent to the larger exponent:

\[
\frac{x^{\frac{1}{2}}y^{\frac{3}{2}}}{x^{\frac{3}{2}}y^{\frac{3}{2}}} = \frac{y^{\frac{3}{2} - \frac{3}{2}}}{x^{\frac{3}{2} - \frac{1}{2}}} = \frac{y^{\frac{3}{2}}}{x^{\frac{1}{2}}} = \frac{y}{x} = x^{-1}y
\]

18. Any number raised to the zeroth power is 1, so \((6.2)^0 = 1\).

20. Use order of operations to simplify the expression inside the parentheses first:

\[
(16^{1/2})^3 = (\sqrt{16})^3 = 4^3 = 64.
\]

22. \(a^4(b^2)^3 + 2ab = (-2)^4(1^2)^3 + 2(-2)(1) = 16 \cdot (1)^3 + (-4) = 16 - 4 = 12\)

24. \((a^{\frac{3}{4}})^2 = \left(\frac{b^3}{a^4}\right)^2 = \frac{b^6}{a^8} = \frac{729}{625}\)

**Scientific Notation**

2a. The decimal place needs to be moved five places to the left:

\[120,000 = 1.2 \times 10^5\]

2b. The decimal place needs to be moved 6 places to the left:

\[1,765,244 = 1.765244 \times 10^6\]

2c. The decimal place needs to be moved 1 place to the left:

\[12 = 1.2 \times 10\]

2d. The decimal place needs to be moved 3 places to the right:

\[0.00281 = 2.81 \times 10^{-3}\]

2e. The decimal place needs to be moved 8 places to the right:

\[0.000000027 = 2.7 \times 10^{-8}\]

4. \((1.60 \times 10^{-19})(6.02 \times 10^{23}) = 9.632 \times 10^4\)

**Exponential Growth Functions**

2. \(y = 5 \cdot 3^t\)
4. \( y = 3 \cdot 10^x \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5/3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>135</td>
</tr>
</tbody>
</table>

6. Step 1: Nadia received $200. A bank is offering an account with 7.5\% interest, compounded yearly. What we want to know: the amount of money in the account by her 21\text{st} birthday.

Step 2: Label variables and set-up equations:

\[ y = A \cdot b^x \]

Define \( y \) as the amount of money in the bank.

Define \( x \) as the number of years from now.

\( A \) is the initial amount, so \( A = 200 \).

Find \( b \). The interest is 7.5\% per year. Change percents to decimals: 7.5\% \( i = 0.075 \). 7.5\% of \( A \) is equal to 0.075\( A \) this represents the interest earned per year.
The total amount of money for the following year: add the interest earned to the initial amount.

\[ A + 0.075A = 1.075A \]

The base of the exponential is \( b = 1.05 \). The formula that describes this problem is:

\[ y = 200(1.075)^x \]

Step 3: To find the total amount of money in the bank at the end of 11 years use \( x = 11 \).

Answer: \( y = 200(1.075)^{11} \approx 443.12 \)

Step 4: Check. The solution checks out.

**Exponential Decay Functions**

2. 

4.  

6a. Start by making a table of values and then draw the graph using the table of values:

<table>
<thead>
<tr>
<th>time (days)</th>
<th>number of bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000000</td>
</tr>
<tr>
<td>1</td>
<td>500000</td>
</tr>
<tr>
<td>2</td>
<td>125000</td>
</tr>
<tr>
<td>3</td>
<td>31250</td>
</tr>
</tbody>
</table>

6b. The exponential decay model fits the pattern:

\[ y = A \cdot b^x \]

\( y \) is the size of the bacteria population
\( x \) is the time measured in days
\( A = 2,000,000 \) is the initial amount
\( b = 1/4 \) since the antibiotic reduces the bacteria population to a quarter of its size each day The formula is:

\[
y = 2000000 \cdot \left( \frac{1}{4} \right)^x \quad \text{or} \quad y = 2000000 \cdot 4^{-x}
\]

6c. To find the size of the bacteria population 10 days after the antibiotic was administered, use \( x = 10 \).

\[
y = 2000000 \cdot \left( \frac{1}{4} \right)^{10} \approx 1.907
\]

Answer: After 10 days, there are essentially 1 or 2 bacteria left.

6d. To find the size of the bacteria population 14 days after the antibiotic was administered, use \( x = 14 \).

\[
y = 2000000 \cdot \left( \frac{1}{4} \right)^{14} \approx 0.0075
\]

Answer: After 14 days, no bacteria are left.

**Geometric Sequences and Exponential Functions**

2.

\[
a_1 = 90 \\
a_2 = 90(-1/3) = -30 \\
a_3 = 90(-1/3)^2 = 10 \\
a_4 = 90(-1/3)^3 = -10/3 \\
a_5 = 90(-1/3)^4 = 10/9
\]

Note that instead of computing higher and higher powers of \(-1/3\), we can get the next term in the sequence by just multiplying the previous by \(-1/3\). For example:

\[
a_3 = a_2(-1/3) = -30(-1/3) = 10.
\]

4. We can find the common ratio using the two consecutive terms:

\[
r = 192/48 = 4.
\]

So the second term is 3(4) = 12, and the last term is 192(4) = 768. Note that since 12(4) = 48, we correctly reproduce the given third term in the sequence.

6. Find the common ratio. We know that to get from \( \frac{9}{4} \) to \( \frac{2}{3} \) in the sequence we must multiply \( \frac{9}{4} \) by the common ratio three times.

\[
\frac{9}{4} r^3 = \frac{2}{3}
\]

Multiply both sides by the reciprocal of \( \frac{9}{4} \):

\[
www.ck12.org \quad 98
\]
\[
r^3 = \frac{4}{9} \cdot \frac{2}{3} = \frac{8}{27}
\]
r is the cube root of \(\frac{8}{27}\).

\[
r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}
\]

To get the 1st missing term, we multiply \(\frac{9}{4}\) by \(r = \frac{2}{3}\) and get: \(\frac{3}{2}\).
To get the 2nd missing term, we multiply \(\frac{3}{2}\) by \(r = \frac{2}{3}\) and get: \(1\).
To check the fourth term, we multiply 1 by \(r = \frac{2}{3}\) and get: \(\frac{2}{3}\). Checks out.
To get the 3rd missing term (the fifth term), we multiply \(\frac{2}{3}\) by \(r = \frac{2}{3}\) and get: \(\frac{4}{9}\).

Answer: The sequence is: \(\frac{9}{4}, \frac{3}{2}, 1, \frac{2}{3}, \frac{4}{9}\).

8. Using the formula for a geometric series, \(a_n = a_1 r^{n-1}\), we have

\[
a_4 = a_1 r^3 = (-7) \left(\frac{3}{4}\right)^3 = (-7) \left(\frac{27}{64}\right) = \frac{189}{64} \approx 2.95.
\]

10. \((Problem\ statement\ incomplete,\ should\ say\ a_5 = 112)\)

To get from \(a_3\) to \(a_5\), we need to multiply by \(r\) twice: \(a_5 = a_3 r^2\). Plugging in values,

\[
112 = 28r^2
4 = r^2
2 = r
\]

Similarly, we find \(a_1\) by dividing by \(r\) twice:

\[
a_1 = a_3 / r^2 = 28/4 = 7.
\]

**Problem-Solving Strategies**

2. Step 1: Read the problem and summarize the information:

In 1990, number of bird species = 1200
Rate of decrease = 1.5% per decade

How many bird species are there left in the year 2020?

Assign variables:
Let \(x\) = time since 1990 (in decades)
Let \(y\) = number of bird species left

Step 2: Look for a pattern:
In 1990: number of bird species = 1200. The rate of decrease is 1.5% (as a decimal 1.5% = 0.015) each decade.

\[
1200 - 1200(0.015) = 1200(1 - 0.015) = 1200(0.985) = 1182
\]
So in 2000: the number of bird species 1182.
In 2010, the number of bird species can be calculated by:

\[ 1182 - 1182(0.015) = 1182(1 - 0.015) = 1182(0.985) = 1164.27 \]

In 2020, the number of bird species can be calculated by:

\[ 1164.27 - 1164.27(0.015) = 1164.27(1 - 0.015) = 1164.27(0.985) = 1146.81 \]

Step 3: We have found the number of bird species left in 2020 in the calculations.
We have found the pattern (1990 = “0”, 2000 = “1”, 2010 = “2”, etc), we multiply by a factor of 0.985.

\[ y = 1200 \cdot (0.985)^x \] where \( x = 0 \) corresponds to 1990, \( x = 1 \) corresponds to 2000, etc.

Answer: According to the model, there will be 1146.81 (round up to 1147) bird species left in 2020 in the given rural area.

Step 4: Check.
To check our answer for the year 2020, plug in \( x = 3 \) (number of decades since 1990)

\[ y = 1200 \cdot (0.985)^3 \approx 1146.81 \]

Looking back over the two solutions we see that we answered the question we were asked and that they make sense.

4. Step 1: Read the problem and summarize the information:
Peter invests $360.
The account pays 7.25% (as a decimal, 7.25% = 0.0725).
The interest is compounded annually (yearly).
What is the total amount in the account after 12 years?
Assign variables:
Let \( x = \) time since money is invested (in years)
Let \( y = \) amount in the account after \( x \) years
Step 2: Look for a pattern:
Year 0: the account has the initial amount = 360
Year 1: the account has earned 7.25% interest and so has a total amount of:

\[ 360 + 360(0.0725) = 360(1 + 0.0725) = 360(1.0725) = 386.10 \]

Year 2: the account has earned 7.25% interest

\[ 386.1 + 386.1(0.0725) = 386.1(1 + 0.0725) = (386.1)(1.0725) \]
\[ (360(1.0725))(1.0725) = 360(1.0725)^2 \approx 414.09 \]

Step 3: We have found the pattern in Step 2:

\[ y = 360(1.0725)^x \]
every year that goes by, we multiply an additional factor of 1.0725.
The total amount in the account after 12 years can be found by setting $x = 12$.

$$y = 360(1.0725)^{12} \approx 833.82$$

Answer: Peter will have a total of $833.82 in the account after 12 years.

Step 4: Check.
The solution checks out.
Chapter 9

TE Factoring Polynomials - Solution Key

9.1 Complete Solutions to Even-Numbered Review Questions

Addition and Subtraction of Polynomials

2. This is a polynomial since it consists of different terms that contain positive, integer powers of the variables \( x \) and \( y \).

4. Since \( \frac{1}{t^2} = t^{-2} \), the expression contains a term with a negative power of \( t \); therefore it is not a polynomial.

6. \( 8 - 4x + 3x^3 = 3x^3 - 4x + 8 \); degree 3 since the highest power of \( x \) appearing is 3.

8. \( x^2 - 9x^4 + 12 = -9x^4 + x^2 + 12 \); degree 4 since the highest power of \( x \) appearing is 4.

10. \((-2x^2 + 4x - 12) + (7x + x^2) = (-2x^2 + x^2) + (4x + 7x) - (12)
    = -x^2 + 11x - 12 \)

12. \((6.9a^2 - 2.3b^2 + 2ab) + (3.1a - 2.5b^2 + b) =
    6.9a^2 + (-2.3b^2 - 2.5b^2) + 2ab + 3.1a + b =
    6.9a^2 - 4.8b^2 + 2ab + 3.1a + b \)

14. \((-y^2 + 4y - 5) - (5y^2 + 2y + 7) =
    (-y^2 + 4y - 5) - 5y^2 - 2y - 7 =
    (-y^2 - 5y^2) + (4y - 2y) + (-5 - 7) =
    -6y^2 + 2y - 12 \)

16. \((2a^2b - 3ab^2 + 5a^2b^2) - (2a^2b^2 + 4a^2b - 5b^2) =
    (2a^2b - 3ab^2 + 5a^2b^2) - 2a^2b^2 - 4a^2b + 5b^2 =
    (5a^2b^2 - 2a^2b^2) + (2a^2b - 4a^2b) - 3ab^2 + 5b^2 =
    3a^2b^2 - 2a^2b - 3ab^2 + 5b^2 \)
18. One red rectangle has area: $a \cdot b = ab$
To find the length of the green rectangle, notice that the “overhang” of the leftmost red rectangle is $c$. So the length of the green rectangle must also be $c$. The width of the green rectangle is clearly $a$.
Therefore the green rectangle has area: $a \cdot c = ac$.
To find the total area of the figure add all the separate areas:
Total area $= ab + ab + ab + ab + ac = 4ab + ac$

20. One way to find the area of this figure is to find the area of the whole rectangle and subtract the area of the new, added rectangle.
The large “completed” rectangle has area: $(b + b)(a + a) = (2b)(2a) = 4ab$
The new, added rectangle has area: $a \cdot b = ab$
To find the total area of the figure subtract:
Total area $= 4ab - ab = 3ab$

**Multiplication of Polynomials**

2. 
$(-5a^2b)(-12a^3b^3) =
(-5)(-12)(a^2 \cdot a^3)(b \cdot b^3) =
60a^5b^4$

3. 
$(3xy^2z^2)(15x^2y^3z) =
(3 \cdot 15)(x \cdot x^2)(y^2 \cdot y)(z^2 \cdot z^3) =
45x^3y^3z^5$

4. 
$2x(4x - 5) = (2x)(4x) - (2x)(5) = 8x^2 - 10x$

6. 
$-3a^2b(9a^2 - 4b^2) =
(9a^2)(-3a^2b) - (4b^2)(-3a^2b) =
-27a^4b + 12a^2b^3$

8. 
$(a^2 + 2)(3a^2 - 4) = (a^2)(3a^2) - (a^2)(4) + (2)(3a^2) - (2)(4) =
3a^4 - 4a^2 + 6a^2 - 8 =
3a^4 + 2a^2 - 8$

10. 
$(2x - 1)(2x^2 - x + 3) =
2x^2(2x - 1) - x(2x - 1) + 3(2x - 1) =
4x^3 - 2x^2 - 2x^2 + x + 6x - 3 =
4x^3 - 4x^2 + 7x - 3$
12. 
\[ (a^2 + 2a - 3)(a^2 - 3a + 4) = \]
\[ a^2(a^2 + 2a - 3) - 3a(a^2 + 2a - 3) + 4(a^2 + 2a - 3) = \]
\[ a^4 + 2a^3 - 3a^2 - 3a^3 - 6a^2 + 9a + 4a^2 + 8a - 12 = \]
\[ a^4 + (2a^3 - 3a^3) + (-3a^2 - 6a^2 + 4a^2) + (9a + 8a) - 12 = \]
\[ a^4 - a^3 - 5a^2 + 17a - 12 \]

14. One way to find the area of this figure is to find the area of the “complete” rectangle and subtract the area of the new, added rectangle.

The large “completed” rectangle has area: \( (x + x)(3x) = (2x)(3x) = 6x^2 \)

The new, added rectangle has area: \( x \cdot (3x - 8) = 3x^2 - 8x \)

To find the total area of the figure subtract:

Total area = \( 6x^2 - (3x^2 - 8x) = 6x^2 - 3x^2 + 8x = 3x^2 + 8x \)

16. The volume of this shape = (area of the base) \( \cdot \) (height)

Area of the base = \( (2x + 4)(3x - 1) = \]
\[ 3x(2x + 4) - (2x + 4) = \]
\[ 6x^2 + 12x - 2x - 4 = \]
\[ 6x^2 + 10x - 4 \]

Height = \( 4x \)

Volume = \( 4x(6x^2 + 10x - 4) = 24x^3 + 40x^2 - 16x \)

**Special Products of Polynomials**

2. If we take \( a = 3x \) and \( b = 7 \), then

\[ (a - b)^2 = a^2 - 2ab + b^2 \Rightarrow \]
\[ (3x - 7)^2 = (3x)^2 - 2(3x)(7) + 7^2 \Rightarrow \]
\[ (3x - 7)^2 = 9x^2 - 42x + 49 \]

4. If we take \( a = 8x \) and \( b = 3 \), then

\[ (a - b)^2 = a^2 - 2ab + b^2 \Rightarrow \]
\[ (8x - 3)^2 = (8x)^2 - 2(8x)(3) + (3)^2 \Rightarrow \]
\[ (8x - 3)^2 = 64x^2 - 48x + 9 \]

6. If we take \( a = x \) and \( b = 12 \), then

\[ (a - b)(a + b) = a^2 - b^2 \Rightarrow \]
\[ (x - 12)(x + 12) = x^2 - (12)^2 \Rightarrow \]
\[ (x - 12)(x + 12) = x^2 - 144 \]

8. If we take \( a = ab \) and \( b = 1 \), then
\[(a - b)(a + b) = a^2 - b^2 \Rightarrow\]
\[(ab - 1)(ab + 1) = (ab)^2 - (1)^2 \Rightarrow\]
\[(ab - 1)(ab + 1) = a^2b^2 - 1\]

10. Rewrite $45 = (50 - 5)$ and $55 = (50 + 5)$. Then

\[45 \times 55 = (50 - 5)(50 + 5) = 50^2 - 5^2 = 2500 - 25 = 2475\]

12. Rewrite $1002 = (1000 + 2)$ and $998 = (1000 - 2)$. Then

\[1002 \times 998 = (1000 + 2)(1000 - 2) = 1000^2 - 2^2 = 1000000 - 4 = 999996\]

**Polynomial Equations in Factored Form**

2. Notice that $5$ and $x^4$ are common factors in each term of the polynomial. Therefore, the monomial factor $5x^4$ is the greatest common factor:

\[5x^6 + 15x^4 = 5x^4(x^2 + 3)\]

4. Notice that $2$ and $x^4$ are common factors in each term of the polynomial. Even $a - 1$ can be considered common to each term. Therefore, the monomial factor $-2x^4$ is the greatest common factor:

\[-10x^6 + 12x^5 - 4x^4 = -2x^4(5x^2 - 6x + 2)\]

6. Notice that $a$ is the only common factor in each term of the polynomial. Therefore, the monomial factor $a$ is the greatest common factor:

\[5a^3 - 7a = a(5a^2 - 7)\]

8. Notice that $4$, $x$ and $y$ are common factors in each term of the polynomial. Therefore, the monomial factor $4xy$ is the greatest common factor:

\[16xy^2z + 4x^3y = 4xy(4yz + x^2)\]

10. Since the polynomial is already factored, set each factor equal to $0$:

\[(2x + 1)(2x - 1) = 0 \Rightarrow\]
\[2x - 1 = 0 \Rightarrow x = 1/2\]
\[2x + 1 = 0 \Rightarrow x = -1/2\]

Answer: The solution set is \(\{1/2, -1/2\}\)

12. Since the polynomial is already factored, set each factor equal to $0$:

\[2x(x + 9)(7x - 20) = 0 \Rightarrow\]
\[2x = 0 \Rightarrow x = 0\]
\[(x + 9) = 0 \Rightarrow x = -9\]
\[(7x - 20) = 0 \Rightarrow x = \frac{20}{7}\]

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Answer: The solution set is: \( \{0, -9, \frac{27}{7}\} \)

14. Write the equation in standard form and factor:

\[
9x^2 = 27x
\]
\[
9x^2 - 27x = 0
\]
\[
9x(x - 3) = 0
\]

Set each factor equal to 0 and solve:

\[
9x = 0 \Rightarrow x = 0
\]
\[
(x - 3) = 0 \Rightarrow x = 3
\]

Answer: The solution set is: \( \{0, 3\} \)

16. Factor:

\[
b^2 - \left(\frac{5}{3}\right)b = 0
\]
\[
b(b - \frac{5}{3}) = 0
\]

Set each factor equal to 0 and solve:

\[
b = 0
\]
\[
b - \frac{5}{3} = 0 \Rightarrow b = \frac{5}{3}
\]

Answer: The solution set is \( \{0, \frac{5}{3}\} \).

**Factoring Quadratic Expressions**

2. We are looking for an answer that is a product of two parentheses:

\[
x^2 + 15x + 50 = (x + _) (x + _)
\]

The number 50 can be written as the product and sum of the following numbers:

\[
50 = 1 \cdot 50 \quad \text{and} \quad 50 + 1 = 51
\]
\[
50 = 2 \cdot 25 \quad \text{and} \quad 25 + 2 = 27
\]
\[
50 = 5 \cdot 10 \quad \text{and} \quad 10 + 5 = 15 \quad \text{This is the correct choice}
\]

Answer: \( x^2 + 15x + 50 = (x + 5)(x + 10) \)

4. We are looking for an answer that is a product of two parentheses:

\[
x^2 + 16x + 48 = (x + _) (x + _)
\]

The number 48 can be written as the product and sum of the following numbers:

\[
48 = 1 \cdot 48 \quad \text{and} \quad 48 + 1 = 49
\]
\[
48 = 2 \cdot 24 \quad \text{and} \quad 24 + 2 = 26
\]
\[
48 = 3 \cdot 16 \quad \text{and} \quad 16 + 3 = 19
\]
\[
48 = 4 \cdot 12 \quad \text{and} \quad 12 + 4 = 16 \quad \text{This is the correct choice}
\]
Answer: \(x^2 + 16x + 48 = (x + 4)(x + 12)\)

6. We are looking for an answer that is a product of two parentheses:

\[x^2 - 13x + 42 = (x - \underline{\phantom{2}})(x - \underline{\phantom{3}})\]

The number 42 can be written as the product and sum of the following numbers:

\[
\begin{align*}
42 &= 1 \cdot 42 & &\text{and} & & 42 + 1 &= 43 \\
42 &= 2 \cdot 21 & &\text{and} & & 21 + 2 &= 23 \\
42 &= 3 \cdot 14 & &\text{and} & & 14 + 3 &= 17 \\
42 &= 6 \cdot 7 & &\text{and} & & 7 + 6 &= 13 & \text{This is the correct choice}
\end{align*}
\]

Answer: \(x^2 - 13x + 42 = (x - 6)(x - 7)\)

8. We are looking for an answer that is a product of two parentheses:

\[x^2 - 9x + 20 = (x - \underline{\phantom{3}})(x - \underline{\phantom{5}})\]

The number 20 can be written as the product and sum of the following numbers:

\[
\begin{align*}
20 &= (-1)(-20) & &\text{and} & & -20 - 1 &= -21 \\
20 &= (-2)(-10) & &\text{and} & & -10 - 2 &= -12 \\
20 &= (-4)(-5) & &\text{and} & & -5 - 4 &= -9 & \text{This is the correct choice}
\end{align*}
\]

Answer: \(x^2 - 9x + 20 = (x - 4)(x - 5)\)

10. We are looking for an answer that is a product of two parentheses:

\[x^2 + 6x - 27 = (x \pm \underline{\phantom{0}})(x \pm \underline{\phantom{0}})\]

The number \(-27\) can be written as the product and sum of the following numbers:

\[
\begin{align*}
-27 &= (-1)(27) & &\text{and} & & 27 - 1 &= 26 \\
-27 &= (1)(-27) & &\text{and} & & -27 + 1 &= -26 \\
-27 &= (3)(-9) & &\text{and} & & -9 + 3 &= -6 \\
-27 &= (-3)(9) & &\text{and} & & 9 - 3 &= 6 & \text{This is the correct choice}
\end{align*}
\]

Answer: \(x^2 + 6x - 27 = (x - 3)(x + 9)\)

12. We are looking for an answer that is a product of two parentheses:

\[x^2 + 4x - 32 = (x \pm \underline{\phantom{0}})(x \pm \underline{\phantom{0}})\]

The number \(-32\) can be written as the product and sum of the following numbers:

\[
\begin{align*}
-32 &= (-1) \cdot 32 & &\text{and} & & -1 + 32 &= 31 \\
-32 &= 1 \cdot (-32) & &\text{and} & & -32 + 1 &= -31 \\
-32 &= (-2) \cdot 16 & &\text{and} & & -2 + 16 &= 14 \\
-32 &= 2 \cdot (-16) & &\text{and} & & -16 + 2 &= -14 \\
-32 &= (-4) \cdot 8 & &\text{and} & & -4 + 8 &= 4 & \text{This is the correct choice}
\end{align*}
\]
Answer: $x^2 + 4x - 32 = (x + 8)(x - 4)$

14. We are looking for an answer that is a product of two parentheses:

$$x^2 - 5x - 50 - (x ± \_)(x ± \_)$$

The number $-50$ can be written as the product and sum of the following numbers:

- $-50 = (-1) \cdot 50$ and $-1 + 50 = 49$
- $-50 = 1 \cdot (-50)$ and $-50 + 1 = -49$
- $-50 = (-2) \cdot 25$ and $-2 + 25 = 23$
- $-50 = 2 \cdot (-25)$ and $-25 + 2 = -23$
- $-50 = (-5) \cdot 10$ and $-5 + 10 = 5$
- $-50 = 5 \cdot (-10)$ and $-10 + 5 = -5$ This is the correct choice

Answer: $x^2 - 5x - 50 = (x - 10)(x + 5)$

16. We are looking for an answer that is a product of two parentheses:

$$x^2 - x - 56 = (x ± \_)(x ± \_)$$

The number $-56$ can be written as the product and sum of the following numbers:

- $-56 = (-1) \cdot 56$ and $-1 + 56 = 55$
- $-56 = 1 \cdot (-56)$ and $-56 + 1 = -55$
- $-56 = (-2) \cdot 28$ and $-2 + 28 = 26$
- $-56 = 2 \cdot (-28)$ and $-28 + 2 = -26$
- $-56 = (-4) \cdot 14$ and $-4 + 14 = 10$
- $-56 = 4 \cdot (-14)$ and $-14 + 4 = -10$
- $-56 = (-7) \cdot 8$ and $-7 + 8 = 1$
- $-56 = 7 \cdot (-8)$ and $-8 + 7 = -1$ This is the correct choice

Answer: $x^2 - x - 56 = (x - 8)(x + 7)$

18. First factor the common factor of $-1$ from each term in the trinomial. Factoring $-1$ just changes the signs of each term in the expression:

$$-x^2 - 5x + 24 = -(x^2 + 5x - 24)$$

We are looking for an answer that is a product of two parentheses:

$$-x^2 - 5x + 24 = -(x^2 + 5x - 24) = -(x ± \_)(x ± \_)$$

The number $-24$ can be written as the product and sum of the following numbers:

- $-24 = (-1)(24)$ and $24 - 1 = 23$
- $-24 = (1)(-24)$ and $-24 + 1 = -23$
- $-24 = (-2)(12)$ and $12 - 2 = 10$
- $-24 = (2)(-12)$ and $-12 + 2 = -10$
- $-24 = (-3)(8)$ and $8 - 3 = 5$ This is the correct choice
Answer: \(-x^2 - 5x + 24 = -(x - 3)(x + 8)\)

20. First factor the common factor of \(-1\) from each term in the trinomial. Factoring \(-1\) just changes the signs of each term in the expression:

\[-x^2 + 25x - 150 = -(x^2 - 25x + 150)\]

We are looking for an answer that is a product of two parentheses:

\[-x^2 + 25x - 150 = -(x^2 - 25x + 150) = -(x - \_\_)(x - \_\_)\]

The number 150 can be written as the product and sum of the following numbers:

- \(150 = (-1)(-150)\) and \(-150 - 1 = -151\)
- \(150 = (-2)(-75)\) and \(-75 - 2 = -77\)
- \(150 = (-3)(-50)\) and \(-50 - 3 = -53\)
- \(150 = (-5)(-30)\) and \(-30 - 5 = -35\)
- \(150 = (-6)(-25)\) and \(-25 - 6 = -31\)
- \(150 = (-10)(-15)\) and \(-10 - 15 = -25\) This is the correct choice

Answer: \(-x^2 + 25x - 150 = -(x - 10)(x - 15)\)

22. First factor the common factor of \(-1\) from each term in the trinomial. Factoring \(-1\) just changes the signs of each term in the expression:

\[-x^2 + 11x - 30 = -(x^2 - 11x + 30)\]

We are looking for an answer that is a product of two parentheses:

\[-x^2 + 11x - 30 = -(x^2 - 11x + 30) = -(x - \_\_)(x - \_\_)\]

The number 30 can be written as the product and sum of the following numbers:

- \(30 = 1 \cdot 30\) and \(30 + 1 = 31\)
- \(30 = 2 \cdot 15\) and \(15 + 2 = 17\)
- \(30 = 3 \cdot 10\) and \(10 + 3 = 13\)
- \(30 = 5 \cdot 6\) and \(6 + 5 = 11\) This is the correct choice

Answer: \(-x^2 + 11x - 30 = -(x - 5)(x - 6)\)

24. We are looking for an answer that is a product of two parentheses:

\[x^2 - 17x - 60 = (x \pm \_\_)(x \pm \_\_)\]

The number \(-60\) can be written as the product and sum of the following numbers:

- \(-60 = (-1) \cdot 60\) and \(-1 + 60 = 59\)
- \(-60 = 1 \cdot (-60)\) and \(-60 + 1 = -59\)
- \(-60 = (-2) \cdot 30\) and \(-2 + 30 = 28\)
- \(-60 = 2 \cdot (-30)\) and \(-30 + 2 = -28\)
- \(-60 = (-3) \cdot 20\) and \(-3 + 20 = 17\)
- \(-60 = 3 \cdot (-20)\) and \(-20 + 3 = -17\) This is the correct choice

Answer: \(x^2 - 17x - 60 = (x - 20)(x + 3)\)
Factoring Special Products

2. Rewrite \(x^2 - 18x + 81\) as \(x^2 - 2(9 \cdot x) + 9^2\). Therefore, \(x^2 - 18x + 81 = (x - 9)^2\)
4. Rewrite \(x^2 + 14x + 49\) as \(x^2 + 2(7 \cdot x) + 7^2\). Therefore, \(x^2 + 14x + 49 = (x - 7)^2\)
6. Rewrite \(25x^2 + 60x + 36\) as \((5x)^2 + 2(5x \cdot 6) + 6^2\). Therefore, \(25x^2 + 60x + 36 = (5x + 6)^2\)
8. Rewrite \(x^4 + 22x^2 + 121\) as \((x^2)^2 + 2(11 \cdot x^2) + 11^2\). Therefore, \(x^4 + 22x^2 + 121 = (x^2 + 11)^2\)
10. Rewrite \(x^2 - 36\) as \((x)^2 - (6)^2\). Therefore, \(x^2 - 36 = (x + 6)(x - 6)\).
12. Rewrite \(x^2 - 400\) as \((x)^2 - (20)^2\). Therefore, \(x^2 - 400 = (x + 20)(x - 20)\).
14. Rewrite \(25x^2 - 49\) as \((5x)^2 - (7)^2\). Therefore, \(25x^2 - 49 = (5x + 7)(5x - 7)\).
16. Rewrite \(16x^2 - 81y^2\) as \((4x)^2 - (9y)^2\). Therefore, \(16x^2 - 81y^2 = (4x + 9y)(4x - 9y)\)
18. Rewrite \(x^2 + 4x = 21\) as \(x^2 + 4x - 21 = 0\). Factor and solve for \(x\):

\[
x^2 + 4x - 21 = 0
\]
\[
(x + 7)(x - 3) = 0
\]
\[
(x + 7) = 0 \Rightarrow x = -7
\]
\[
(x - 3) = 0 \Rightarrow x = 3
\]

Answer: The solution set is: \([-7, 3]\)

20. Factor and solve for \(x\):

\[
x^2 - 64 = 0
\]
\[
(x + 8)(x - 8) = 0
\]
\[
x + 8 = 0 \Rightarrow x = -8
\]
\[
x - 8 = 0 \Rightarrow x = 8
\]

Answer: The solution set is \([8, -8]\).

22. Factor and solve for \(x\):

\[
4x^2 - 25 = 0
\]
\[
(2x)^2 - (5)^2 = 0
\]
\[
(2x + 5)(2x - 5) = 0
\]
\[
2x + 5 = 0 \Rightarrow x = -5/2
\]
\[
2x - 5 = 0 \Rightarrow x = 5/2
\]

Answer: The solution set is \([5/2, -5/2]\).

24. Rewrite \(-x^2 - 16x - 60 = 0\) as \(-(x^2 + 16x + 60) = 0\). Factor and solve for \(x\):

\[
-(x^2 + 16x + 60) = 0
\]
\[
-(x + 6)(x + 10) = 0
\]
\[
-(x + 6) = 0 \Rightarrow x = -6
\]
\[
(x + 10) = 0 \Rightarrow x = -10
\]

Answer: The solution set is: \([-6, -10]\)

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Factoring Polynomials Completely

2. Factor the common monomial. In this case factor \(-x\) rather than \(x\). It is always easier to factor the negative number so that the leading term is positive.

\[
-x^3 + 17x^2 - 70x = \\
-x(x^2 - 17x + 70) = \\
-x(x - 7)(x - 10)
\]

4. Factor the common monomial. In this expression, it is \(3x\).

\[
12x^3 + 12x^2 + 3x = \\
3x(4x^2 + 4x + 1) = \\
3x(2x + 1)(2x + 1) = \\
3x(2x + 1)^2
\]

6. There is no factor that is common to all the terms. However, there is a factor of \(5x\) common to the first two terms. Factor \(5x\) from the first two terms:

\[
5x^2 - 35x + x - 7 = 5x(x - 7) + x - 7
\]

Now we notice that the binomial \((x - 7)\) is common to both terms. We factor the common binomial and get:

\[
5x^2 - 35x + x - 7 = 5x(x - 7) + x - 7 = 5x(x - 7) + (x - 7) = 5x(x - 7) + 1 \cdot (x - 7) = (x - 7)(5x + 1)
\]

Answer: \(5x^2 - 35x + x - 7 = (x - 7)(5x + 1)\)

8. There is no factor that is common to all the terms. However, there is a factor of \(4x\) common to the first two terms and a factor of \(-5\) common to the last two terms. Factor \(4x\) from the first two terms and \(-5\) from the last two:

\[
4x^2 + 32x - 5x - 40 = 4x(x + 8) - 5(x + 8)
\]

Now we notice that the binomial \((x + 8)\) is common to both terms. We factor the common binomial and get:

\[
4x^2 + 32x - 5x - 40 = 4x(x + 8) - 5(x + 8) = (x + 8)(4x - 5)
\]

Answer: \(4x^2 + 32x - 5x - 40 = (x + 8)(4x - 5)\)

10. Rewrite \(6x^2 + 7x + 1\) as \((6x^2 + 6x) + (x + 1)\). Now factor out \((x + 1)\) from each term:

\[
(6x^2 + 6x) + (x + 1) = 6x(x + 1) + 1(x + 1) \\
= (6x + 1)(x + 1).
\]

12. Rewrite \(3x^2 + 16x + 21\) as \((3x^2 + 9x) + (7x + 21)\) (since the only factors of 21 are 3 and 7, we knew we wanted to get either a 3x or a 7x in the last term in order to factor by grouping). Now factor out \((x + 3)\) from each term:
\[(3x^2 + 9x) + (7x + 21) = 3x(x + 3) + 7(x + 3)\]
\[= (x + 3)(3x + 7).\]

14. Step 1: What we know: The rectangle has an area of 108, with sides of \(x + 2\) and \(x - 1\).
What we want to know: the value of \(x\)
Step 2: Label variables and set-up equations.
Fortunately, the variable is already labeled here!
Use the formula for the area of a rectangle:
\[\text{Area} = (\text{Length})(\text{Width}) = (x + 2)(x - 1)\]
Simplifying and setting the area equal to 108 results in:
\[x^2 + x - 110 = 0\]
Step 3: Solve.
\[(x + 11)(x - 10) = 0\]
\[x + 11 = 0 \Rightarrow x = -11; \text{ this is NOT allowed since this would make both of the sides have negative length}\]
\[x - 10 = 0 \Rightarrow x = 10\]
Answer: \(x = 10\)
Step 4: Plugging in the value of \(x\), we have side lengths of 12 and 9, which multiply to give an area of 108.
The answer checks out.

16. Step 1: What we know: Cost of glass = $1/sq ft. Cost of frame = $2/sq ft. The frame is to be square.
What we want to know: the size (length or width) of the frame for $20.
Step 2: Label variables and set-up equations.
Let \(x\) = the length (and width, since the picture is to be square) of the frame.
Then (drawing a picture helps!):
\[\text{Total Cost} = \text{Cost of the glass} + \text{Cost of the frame} = 1 \cdot x^2 + 2 \cdot (4x)\]
Simplifying and setting this cost equation equal to 20 results in:
\[x^2 + 8x = 20\]
Step 3: Solve.
\[x^2 + 8x - 20 = 0\]
\[(x + 10)(x - 2) = 10\]
\[(x + 10) = 0 \Rightarrow x = -10\]
\[(x - 2) = 0 \Rightarrow x = 2\]
Answer: You should ask for a 2 ft by 2 ft frame.
Step 4: The solution checks out.
Chapter 10

TE Quadratic Equations and Quadratic Functions - Solution Key

10.1 Complete Solutions to Even-Numbered Review Questions

Graphs of Quadratic Functions

2. Write the quadratic function in intercept form by factoring the right hand side of the equation:

\[ y = -x^2 + 10x - 21 \]
\[ y = -(x^2 - 10x + 21) \]
\[ y = -(x - 7)(x - 3) \]

The function in intercept form is: \[ y = -(x - 7)(x - 3) \]

We find the \( x \)-intercepts by setting \( y = 0 \).

We have:

\[ 0 = -(x - 7)(x - 3) \]
\[ -(x - 7) = 0 \Rightarrow x = 7 \]
\[ (x - 3) = 0 \Rightarrow x = 3 \]

So, the \( x \)-intercepts are: \((7, 0)\) and \((3, 0)\).

The vertex is halfway between the two \( x \)-intercepts. We find the \( x \)-value by taking the average of the two \( x \)-intercepts:

\[ x = \frac{7 + 3}{2} = 5 \]

We find the \( y \)-value by plugging the \( x \)-value we just found in the original equation:
\[ y = -5^2 + 10(5) - 21 = -25 + 50 - 21 = 4 \]

So, the vertex is: \((5, 4)\).

4. Down, since the coefficient of \(x^2\) is negative.

6. Down, since the coefficient of \(x^2\) is negative.

8. Comparing equations \(y = -2x^2\) and \(y = -2x^2 - 2\), we see that the constant term \(-2\) shifts the vertex of the parabola represented by the equation \(y = -2x^2 - 2\) two units lower that that of \(y = -2x^2\).

This can be confirmed by finding the vertices of each parabola. Each vertex will have an \(x\)-coordinate of 0. To find \(y\), replace the \(x\)'s in the original equations with 0 to obtain the vertices \((0, 0)\) and \((0, -2)\), respectively.

Answer: The vertex of the parabola represented by \(y = -2x^2\) is higher.

10. The larger the coefficient of the second degree term, the skinnier the graph is. Therefore, the parabola represented by \(y = x^2\) is wider than that of \(y = 4x^2\).

12. The larger the coefficient of the second degree term (in absolute value), the skinnier the graph is. Therefore, the parabola represented by \(y = -x^2 - 2\) is wider than that of \(y = -2x^2 - 2\).

14. The vertex and \(x\)-intercepts are found to help you pick values for the table.

\[
\begin{align*}
y & = -x^2 + x + 12 \\
y & = -(x^2 - x - 12) \\
y & = -(x - 4)(x + 3) \\
0 & = -(x - 4)(x + 3) \\
(x - 4) & = 0 \Rightarrow x = 4 \\
(x + 3) & = 0 \Rightarrow x = -3
\end{align*}
\]

The two \(x\)-intercepts are \(x = 4\) and \(x = -3\). The \(x\)-coordinate of the vertex is the average of the two intercepts:

\[
x = \frac{4 + (-3)}{2} = \frac{1}{2}
\]

16. The vertex and \(x\)-intercepts are found to help you pick values for the table.

\[
\begin{align*}
y & = \frac{1}{2}x^2 - 2x \\
y & = \frac{1}{2}x(x - 4) \\
0 & = \frac{1}{2}x(x - 4) \\
x & = 0 \\
(x - 4) & = 0 \Rightarrow x = 4
\end{align*}
\]

The two \(x\)-intercepts are \(x = 0\) and \(x = 4\). The \(x\)-coordinate of the vertex is the average of the two intercepts:

\[
x = \frac{4 + 0}{2} = 2
\]
18. The vertex and $x$–intercepts are found to help you pick values for the table.

$$y = 4x^2 - 8x + 4$$
$$y = 4(x^2 - 2x + 1)$$
$$y = 4(x - 1)(x - 1)$$
$$0 = 4(x - 1)(x - 1)$$
$$(x - 1) = 0 \Rightarrow x = 1$$

The $x$–intercept is $x = 1$. Because there is only one $x$–intercept, the parabola must be situated so that its vertex is on the $x$–axis; that is, the vertex is the $x$–intercept.

It can be helpful to find the $y$–intercept:

$$y = 4x^2 - 8x + 4$$
$$y = 4(0)^2 - 8(0) + 4 = 4$$

Therefore the $y$–intercept is $y = 4$.

20. The greatest area corresponds to the maximum $y$–value of the graph; since this parabola opens downwards, it has a maximum at the vertex. Looking at the graph, we see that the vertex is at $x = 30$ feet, so the length is $120 - 2x = 60$ feet, and the rectangle is 30 feet by 60 feet.

**Quadratic Equations by Graphing**

2. The quadratic intersects the $x$–axis at $x = -1.2$ and $x = 1.87$, so these are the solutions.
4. The quadratic only intersects the $x$–axis once at $x = -3$, so $-3$ is a double root and the only solution. Note that $x^2 + 6x + 9$ takes the form of a special quadratic, so we can factor it and see the solutions directly: $x^2 + 6x + 9 = (x + 3)^2$.

6. The quadratic never intersects the $x$–axis, so there are no real solutions.
8. The quadratic intersects the $x$–axis at $x = -1.5$ and $x = 1.5$, so these are the solutions. Note that $y = 9 - 4x^2$ takes the form of a special quadratic, so we can factor it and see the solutions directly: $9 - 4x^2 = (3)^2 - (2x)^2 = (3 + 2x)(3 - 2x)$.

10. The quadratic only intersects the $x$–axis once at $x = -5$, so $-5$ is a double root and the only solution. Note that $-x^2 - 10x - 25$ takes the form of a special quadratic, so we can factor it and see the solutions directly: $-x^2 - 10x - 25 = -(x + 5)^2$. 
12. The quadratic never intersects the $x$–axis, so there are no real solutions.

14. Using a graphing calculator, we see that the quadratic never intersects the $x$–axis, so there are no real roots. The vertex is at $(-1.5, 3.75)$.

16. The ball reaches the ground when the height is 0. Setting $y = 0$ in the equation and solving results in

\[
y = -16t^2 + 60t
\]

\[
0 = -16t^2 + 60t
\]

\[
\Rightarrow t = 0 \text{ seconds } \text{ or } t = 3.75 \text{ seconds}
\]

The first root says that at time 0 seconds the height of the ball is 0 meters. The second root says that it takes approximately 3.75 seconds for the ball to return to the ground. See graph provided in the answers to the Review Questions.
Quadratic Equations by Square Roots

2.

\[ x^2 - 100 = 0 \]
\[ x^2 = 100 \]
\[ \sqrt{x^2} = \sqrt{100} \]
\[ x = \pm 10 \]

Also note the difference of squares on the left side of the equation: \( x^2 - 100 = 0 \). Since \( x^2 - 100 = (x + 10)(x - 10) \), we can confirm our calculation above:

\( (x + 10) = 0 \Rightarrow x = -10 \)
\( (x - 10) = 0 \Rightarrow x = 10 \)

4.

\[ 9x^2 - 1 = 0 \]
\[ 9x^2 = 1 \]
\[ x^2 = \frac{1}{9} \]
\[ \sqrt{x^2} = \frac{1}{\sqrt{9}} \]
\[ x = \pm \frac{1}{3} \]

Also note the difference of squares on the left side of the equation: \( 9x^2 - 1 = 0 \). Since \( 9x^2 - 1 = (3x+1)(3x-1) \), we can confirm our calculation above:

\( (3x + 1) = 0 \Rightarrow x = -\frac{1}{3} \)
\( (3x - 1) = 0 \Rightarrow x = \frac{1}{3} \)

6.

\[ 64x^2 - 9 = 0 \]
\[ 64x^2 = 9 \]
\[ x^2 = \frac{9}{64} \]
\[ \sqrt{x^2} = \frac{\sqrt{9}}{\sqrt{64}} \]
\[ x = \pm \frac{3}{8} \]

Also note the difference of squares on the left side of the equation: \( 64x^2 - 9 = 0 \). Since \( 64x^2 - 9 = (8x + 3)(8x - 3) \), we can confirm our calculation above:

\( (8x + 3) = 0 \Rightarrow x = -\frac{3}{8} \)
\( (8x - 3) = 0 \Rightarrow x = \frac{3}{8} \)

8.
Also note the difference of squares on the left side of the equation: $25x^2 - 36 = 0$. Since $25x^2 - 36 = (5x + 6)(5x - 6)$, we can confirm our calculation above:

\[
(5x + 6) = 0 \Rightarrow x = -\frac{6}{5}
\]
\[
(8x - 3) = 0 \Rightarrow x = \frac{3}{5}
\]

10.

\[
x^2 - 16 = 0
\]
\[
x^2 = 16
\]
\[
\sqrt{x^2} = \sqrt{16}
\]
\[
x = \pm 4
\]

Also note the difference of squares on the left side of the equation: $x^2 - 16 = 0$. Since $x^2 - 16 = (x + 4)(x - 4)$, we can confirm our calculation above:

\[
(x + 4) = 0 \Rightarrow x = -4
\]
\[
(x - 4) = 0 \Rightarrow x = 4
\]

12.

\[
16x^2 - 49 = 0
\]
\[
16x^2 = 49
\]
\[
x^2 = \frac{16}{49}
\]
\[
\sqrt{x^2} = \sqrt{\frac{16}{49}}
\]
\[
x = \pm \frac{4}{7}
\]

Also note the difference of squares on the left side of the equation: $16x^2 - 49 = 0$. Since $16x^2 - 49 = (4x + 7)(4x - 7)$, we can confirm our calculation above:

\[
(4x + 7) = 0 \Rightarrow x = -\frac{7}{4}
\]
\[
(4x - 7) = 0 \Rightarrow x = \frac{7}{4}
\]

14.

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\( (x + 5)^2 = 16 \)
\[ \sqrt{(x + 5)^2} = \sqrt{16} \]
\[ x + 5 = \pm 4 \]
\[ x = -5 \pm 4 \]
\[ \Rightarrow x = -5 + 4 = -1 \]
\[ \Rightarrow x = -5 - 4 = -9 \]

Answers: The roots are \(-1\) and \(-9\)

16. \( (3x + 4)^2 = 9 \)
\[ \sqrt{(3x + 4)^2} = \sqrt{9} \]
\[ 3x + 4 = \pm 3 \]
\[ 3x = -4 \pm 3 \]
\[ x = \frac{-4 \pm 3}{3} \]
\[ \Rightarrow x = \frac{-4 + 3}{3} = -1 \]
\[ \Rightarrow x = \frac{-4 - 3}{3} = -3 \]

Answers: The roots are \(-1\) and \(-3\).

18. \( x^2 - 6 = 0 \)
\[ x^2 = 6 \]
\[ \sqrt{x^2} = \sqrt{6} \]
\[ x = \pm \sqrt{6} \]

20. \( 3x^2 + 14 = 0 \)
\[ 3x^2 = -14 \]
\[ x^2 = -\frac{14}{3} \]

No square of any real number is negative; therefore there are no real solutions to the equation.

22. \( (4x + 1)^2 - 8 = 0 \)
\[ (4x + 1)^2 = 8 \]
\[ \sqrt{(4x + 1)^2} = \sqrt{8} \]
\[ 4x + 1 = \pm 2 \sqrt{2} \]
\[ 4x = -1 \pm 2 \sqrt{2} \]
\[ x = \frac{-1 \pm 2 \sqrt{2}}{4} \]
\[ \Rightarrow x = \frac{-1 + 2 \sqrt{2}}{4} \approx 0.4571 \]
\[ \Rightarrow x = \frac{-1 - 2 \sqrt{2}}{4} \approx 0.9571 \]
24. 
\[x^2 + 18x + 81 = 1\]
\[x^2 + 18x + 80 = 0\]
\[(x + 10)(x + 8) = 0\]
\[(x + 10) = 0 \Rightarrow x = -10\]
\[(x + 8) = 0 \Rightarrow x = -8\]

26. 
\[(x + 10)^2 = 2\]
\[\sqrt{(x + 10)^2} = \sqrt{2}\]
\[x + 10 = \pm \sqrt{2}\]
\[x = -10 \pm \sqrt{2}\]
\[\Rightarrow x = -10 + \sqrt{2} \approx -8.59\]
\[\Rightarrow x = -10 - \sqrt{2} \approx -11.41\]

28. 
\[2(x + 3)^2 = 8\]
\[(x + 3)^2 = 4\]
\[\sqrt{(x + 3)^2} = \sqrt{4}\]
\[x + 3 = \pm 2\]
\[x = -3 \pm 2\]
\[\Rightarrow x = -3 + 2 = -1\]
\[\Rightarrow x = -3 - 2 = -5\]

Answers: The roots are -1 and -5.

30. Since we want the height in meters, use equation: \[y = -4.9t^2 + y_0\]
The time of flight is \(t = 5.3\) seconds: \[y = -4.9(5.3)^2 + y_0\]
The height when the ball hits the ground is \(y = 0\), so: \(0 = -4.9(5.3)^2 + y_0\)
Simplify: \(0 = -137.6 + y_0\) so \(y_0 = 137.6\)
Answer: The cliff is 137.6 meters high.

**Quadratic Equations by Completing the Square**

2. We complete the square by adding the constant term 1: \((x^2 - 2x) + 1 = (x - 1)^2\)

4. We complete the square by adding the constant term 4: \((x^2 - 4x) + 4 = (x^2 - 2(2)x) + 2^2 = (x - 2)^2\)

6. Factor the coefficient of the \(x^2\) term: \(2(x^2 - 11x)\). Now complete the square of the expression in parentheses:
Re-write the expression: \(2\left(x^2 - 2\left(\frac{11}{2}\right)x\right)\)

We complete the square by adding the constant \(\left(\frac{11}{2}\right)^2\): \(2\left(x^2 - 2\left(\frac{11}{2}\right)x + \left(\frac{11}{2}\right)^2\right)\)

Factor the perfect square trinomial inside the parenthesis:
Answer: $2 \left( x - \frac{1}{2} \right)^2$

8. Factor the coefficient of the $x^2$ term: $5 \left( x^2 + \frac{12}{5} x \right)$. Now complete the square of the expression in parentheses:

Re-write the expression: $5 \left( x^2 + 2 \left( \frac{6}{5} \right) x \right)$

We complete the square by adding the constant $\left( \frac{6}{5} \right)^2$: $5 \left( x^2 + 2 \left( \frac{6}{5} \right) x + \left( \frac{6}{5} \right)^2 \right)$

Factor the perfect square trinomial inside the parenthesis:

Answer: $5 \left( x + \frac{6}{5} \right)^2$

10. Rewrite as:

$$x^2 - 2 \left( \frac{5}{2} \right) x = 10$$

Add the constant to both sides of the equation:

$$x^2 - 2 \left( \frac{5}{2} \right) x + \left( \frac{5}{2} \right)^2 = 10 + \left( \frac{5}{2} \right)^2$$

Factor the perfect square trinomial and simplify.

$$\left( x - \frac{5}{2} \right)^2 = 10 + \left( \frac{5}{2} \right)^2$$

$$\left( x - \frac{5}{2} \right)^2 = \frac{40}{4} + \frac{25}{4}$$

$$\left( x - \frac{5}{2} \right)^2 = \frac{65}{4}$$

Take the square root of both sides:

$$\sqrt{\left( x - \frac{5}{2} \right)^2} = \sqrt{\frac{65}{4}}$$

$$x - \frac{5}{2} = \pm \sqrt{\frac{65}{2}}$$

$$x = \frac{5}{2} \pm \sqrt{\frac{65}{2}}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$\Rightarrow x = \frac{5 + \sqrt{65}}{2} \approx 6.53$$

$$\Rightarrow x = \frac{5 - \sqrt{65}}{2} \approx -1.53$$

Answer: $x = 6.53$ or $x = -1.53$

12. Move the constant to the other side of the equation:

$$x^2 + 15x = -20$$

Rewrite as:
\[ x^2 + 2\left(\frac{15}{2}\right)x = -20 \]

Add the constant to both sides of the equation:

\[ x^2 + 2\left(\frac{15}{2}\right)x + \left(\frac{15}{2}\right)^2 = -20 + \left(\frac{15}{2}\right)^2 \]

Factor the perfect square trinomial and simplify.

\[ \left(x + \frac{15}{2}\right)^2 = -20 + \left(\frac{15}{2}\right)^2 \]
\[ \left(x + \frac{15}{2}\right)^2 = -\frac{80}{4} + \frac{225}{4} \]
\[ \left(x + \frac{15}{2}\right)^2 = \frac{145}{4} \]

Take the square root of both sides:

\[ \sqrt{\left(x + \frac{15}{2}\right)^2} = \sqrt{\frac{145}{4}} \]
\[ x + \frac{15}{2} = \pm \frac{\sqrt{145}}{2} \]
\[ x = -\frac{15}{2} \pm \frac{\sqrt{145}}{2} \]

\[ x = -15 \pm \sqrt{145} \]
\[ \Rightarrow x = \frac{-15 + \sqrt{145}}{2} \approx -1.48 \]
\[ \Rightarrow x = \frac{-15 - \sqrt{145}}{2} \approx -13.52 \]

Answer: \( x = -1.48 \) and \( x = -13.52 \)

14. Factor out the constant term:

\[ 4\left(x^2 + \frac{5}{4}x\right) = -1 \]

Divide both sides by 4 and rewrite as:

\[ x^2 + 2\left(\frac{5}{8}\right)x = -\frac{1}{4} \]

Add the constant to both sides of the equation:

\[ x^2 + 2\left(\frac{5}{8}\right)x + \left(\frac{5}{8}\right)^2 = -\frac{1}{4} + \left(\frac{5}{8}\right)^2 \]

Factor the perfect square trinomial and simplify.
\[
(x + \frac{5}{8})^2 = -\frac{1}{4} + \left(\frac{5}{8}\right)^2
\]
\[
(x + \frac{5}{8})^2 = \frac{-16}{64} + \frac{25}{64}
\]
\[
(x + \frac{5}{8})^2 = \frac{9}{64}
\]

Take the square root of both sides:

\[
\sqrt{(x + \frac{5}{8})^2} = \sqrt{\frac{9}{64}}
\]
\[
x + \frac{5}{8} = \pm \frac{3}{8}
\]
\[
x = \frac{5 \pm 3}{8}
\]
\[
\Rightarrow x = \frac{-5 + 3}{8} = \frac{-2}{8} = \frac{-1}{4}
\]
\[
\Rightarrow x = \frac{-5 - 3}{8} = -1
\]

Answer: \(x = -1/4\) and \(x = 1\)

16. Move the constant to the other side of the equation:

\[5x^2 + 15x = 40\]

Rewrite as:

\[x^2 + 3x = 8\]
\[x^2 + 2\left(\frac{3}{2}\right)x = 8\]

Add the constant to both sides of the equation:

\[x^2 + 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 = 8 + \left(\frac{3}{2}\right)^2\]

Factor the perfect square trinomial and simplify.

\[\left(x + \frac{3}{2}\right)^2 = 8 + \left(\frac{3}{2}\right)^2\]
\[\left(x + \frac{3}{2}\right)^2 = \frac{32}{4} + \frac{9}{4}\]
\[\left(x + \frac{3}{2}\right)^2 = \frac{41}{4}\]

Take the square root of both sides:
\[
\sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{41}{4}}
\]
\[
x + \frac{3}{2} = \pm \frac{\sqrt{41}}{2}
\]
\[
x = -\frac{3}{2} \pm \frac{\sqrt{41}}{2}
\]
\[
x = \frac{-3 \pm \sqrt{41}}{2}
\]
\[
\Rightarrow x = \frac{-3 + \sqrt{41}}{2} \approx 1.7
\]
\[
\Rightarrow x = \frac{-3 - \sqrt{41}}{2} \approx -4.7
\]

Answer: \(x = 1.7\) and \(x = -4.7\)

18. Complete the square first:

\[
y + 1 = -2x^2 - x
\]
\[
y + 1 = -2\left(x^2 + \frac{1}{2}x\right)
\]
\[
y + 1 = -2\left(x^2 + 2\left(\frac{1}{4}\right)x\right)
\]
\[
y + 1 - 2\left(\frac{1}{4}\right)^2 = -2\left(x^2 + 2\left(\frac{1}{4}\right)x + \left(\frac{1}{4}\right)^2\right)
\]
\[
y + 1 - 2\left(\frac{1}{16}\right)^2 = -2\left(x + \frac{1}{4}\right)^2
\]
\[
y + 1 - \frac{1}{8} = -2\left(x + \frac{1}{4}\right)^2
\]
\[
y + \frac{7}{8} = -2\left(x + \frac{1}{4}\right)^2
\]
\[
y - \left(\frac{7}{8}\right) = -2\left(x - \left(-\frac{1}{4}\right)^2\right)
\]

20. Complete the square first:

\[
y = -32x^2 + 60x + 10
\]
\[
y - 10 = -32\left(x^2 - \frac{60}{32}x\right)
\]
\[
y - 10 = -32\left(x^2 - 2\left(\frac{30}{32}\right)x\right)
\]
\[
y - 10 = -32\left(x^2 - 2\left(\frac{15}{16}\right)x\right)
\]
\[
y - 10 - 32\left(\frac{15}{16}\right)^2 = -32\left(x^2 - 2\left(\frac{15}{16}\right)x + \left(\frac{15}{16}\right)^2\right)
\]
\[
y - 10 - 32\left(\frac{15}{16}\right)^2 = -32\left(x - \frac{15}{16}\right)^2
\]
\[
y - \frac{305}{8} = -32\left(x - \frac{15}{16}\right)^2
\]
22a. Complete the square first:

\[ y = -4x^2 + 20x - 24 \]
\[ y + 24 = -4(x^2 - 5x) \]
\[ y + 24 = -4\left(x^2 - 2\left(\frac{5}{2}\right)x\right) \]
\[ y + 24 - 4\left(\frac{5}{2}\right)^2 = -4\left(x^2 - 2\left(\frac{5}{2}\right)x + \left(\frac{5}{2}\right)^2\right) \]
\[ y - 1 = -4\left(x - \frac{5}{2}\right)^2 \]

Therefore, the vertex is at \( \left(\frac{5}{2}, 1\right) \)

22b. To find the \( x \)-intercepts, set \( y = 0 \).

\[ y - 1 = -4\left(x - \frac{5}{2}\right)^2 \]
\[ 0 - 1 = -4\left(x - \frac{5}{2}\right)^2 \]
\[ \frac{1}{4} = \left(x - \frac{5}{2}\right)^2 \]
\[ \sqrt{\frac{1}{4}} = \sqrt{\left(x - \frac{5}{2}\right)^2} \]
\[ \pm \frac{1}{2} = x - \frac{5}{2} \]
\[ x = \frac{5}{2} \pm \frac{1}{2} \]
\[ \Rightarrow x = \frac{5}{2} + \frac{1}{2} = 3 \]
\[ \Rightarrow x = \frac{5}{2} - \frac{1}{2} = 3 \]

Therefore the \( x \)-intercepts are 2 and 3.

22c. To find the \( y \)-intercept, set \( x = 0 \).

\[ y = -4x^2 + 20x - 24 \]
\[ y = -4(0)^2 + 20(0) - 24 \]
\[ y = -24 \]

Therefore the \( y \)-intercept is \(-24\).

22d. Since the coefficient of the squared term is negative, the parabola turns down.
24a. Complete the square first:

\[ y + 6 = -x^2 + x \]
\[ y + 6 = -(x^2 - 2\left(\frac{1}{2}\right)x) \]
\[ y + 6 - \left(\frac{1}{2}\right)^2 = -(x^2 - 2\left(\frac{1}{2}\right)x + \left(\frac{1}{2}\right)^2) \]
\[ y + 6 - \frac{1}{4} = -(x - \frac{1}{2})^2 \]
\[ y + \frac{23}{4} = -(x - \frac{1}{2})^2 \]

Therefore, the vertex is at \( \left(\frac{1}{2}, \frac{23}{4}\right) \)

24b. To find the \( x \)-intercepts, set \( y = 0 \).

\[ y + \frac{23}{4} = -(x - \frac{1}{2})^2 \]
\[ 0 + \frac{23}{4} = -(x - \frac{1}{2})^2 \]
\[ -\frac{23}{4} = -(x - \frac{1}{2})^2 \]
\[ \frac{23}{4} = (x - \frac{1}{2})^2 \]

Since no real number is the square of a negative number, the equation has no real roots, and thus no \( x \)-intercepts.

24c. To find the \( y \)-intercept, set \( x = 0 \).

\[ y + 6 = -x^2 + x \]
\[ y + 6 = -0^2 + 0 \Rightarrow y = -6 \]

Therefore the \( y \)-intercept is \(-6\).

24d. Since the coefficient of the squared term is negative, the parabola turns down.
26. Drawing a picture helps.

Let $x =$ the distance Amanda has walked.

Then $x + 3 =$ the distance Dolvin has biked.

Since Amanda and Dolvin left in perpendicular directions, we can use the Pythagorean Theorem to write an equation in terms of $x$:

$$x^2 + (x + 3)^2 = 5.5^2$$

Expand and simplify:

$$x^2 + x^2 + 6x + 9 = 30.25$$

$$2x^2 + 6x - 21.25 = 0$$

Complete the square to solve for $x$:  

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Since only positive distances make sense here, the distance Amanda has walked is 2.09 miles and the distance Dolvin has biked is $3 + 2.09 = 5.09$ miles.

## Quadratic Equations by the Quadratic Formula

2. Rewrite the equation in standard form:

$$x^2 - 6x - 12 = 0$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plug in the values $a = 1, b = -6, c = -12$

$$x = \frac{-(6) \pm \sqrt{(-6)^2 - 4(1)(-12)}}{2(1)}$$

Simplify:
\[ x = \frac{6 \pm \sqrt{36 + 48}}{2} \]
\[ x = \frac{6 \pm \sqrt{84}}{2} \]
\[ x = \frac{6 \pm 2\sqrt{21}}{2} = 3 \pm \sqrt{21} \]
\[ \Rightarrow x = 3 + \sqrt{21} \approx 7.58 \]
\[ \Rightarrow x = 3 - \sqrt{21} \approx -1.58 \]

Answer: \( x = 7.58 \) or \( x = -1.58 \)

4. The equation is already in standard form:

\[ 2x^2 + x - 3 = 0 \]

Quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Plug in the values \( a = 2, b = 1, c = -3 \)

\[ x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-3)}}{2(2)} \]

Simplify:

\[ x = \frac{-1 \pm \sqrt{1 + 24}}{4} \]
\[ x = \frac{-1 \pm 5}{4} \]
\[ \Rightarrow x = \frac{-1 + 5}{4} = 1 \]
\[ \Rightarrow x = \frac{-1 - 5}{4} = -1.5 \]

Answer: \( x = 1 \) or \( x = -1.5 \)

6. The equation is already in standard form:

\[ -3x^2 + 5x = 0 \]

Quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Plug in the values \( a = -3, b = 5, c = 0 \)

\[ x = \frac{-5 \pm \sqrt{5^2 - 4(-3)(0)}}{2(-3)} \]
Simplify:

\[ x = \frac{-5 \pm 5}{-6} \]

\[ \Rightarrow x = \frac{-5 + 5}{-6} = 0 \]

\[ \Rightarrow x = \frac{-5 - 5}{-6} = \frac{5}{3} \]

Answer: \( x = 0 \) or \( x = \frac{5}{3} \) (note SE gives incorrect solutions \( x = 1 \) and \( x = \frac{2}{3} \))

8. Rewrite the equation in standard form:

\[ x^2 + 2x + 6 = 0 \]

Quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Plug in the values \( a = 1, b = 2, c = 6 \)

\[ x = \frac{-2 \pm \sqrt{2^2 - 4(1)(6)}}{2(1)} \]

Simplify:

\[ x = \frac{-2 \pm \sqrt{-20}}{2} \]

Answer: No real solutions.

10. Because the middle term is missing, solve by square-roots:

\[ x^2 - 12 = 0 \]

\[ x^2 = 12 \]

\[ \sqrt{x^2} = \sqrt{12} \]

\[ x = \pm \sqrt{12} \]

\[ x = \pm 2 \sqrt{3} \]

12. A moment’s thought shows that the equation can be factored over the integers:

\[ x^2 + 7x - 18 = 0 \]

\[ (x + 9)(x - 2) = 0 \]

\[ (x + 9) = 0 \Rightarrow x = -9 \]

\[ (x - 2) = 0 \Rightarrow x = 2 \]

14. Reduce the equation by \(-4\) first, then either complete the square or use the quadratic formula:

\[ -4x^2 + 4000x = 0 \]

\[ x^2 - 1000x = 0 \]
Quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Plug in the values \(a = 1, b = -1000, c = 0\)

\[ x = \frac{-1000 \pm \sqrt{(-1000)^2 - 4(1)(0)}}{2(1)} \]

Simplify:

\[ x = \frac{-1000 \pm \sqrt{(-1000)^2}}{2} \]
\[ x = \frac{-1000 \pm 1000}{2} \]
\[ \Rightarrow x = \frac{-1000 + 1000}{2} = 0 \]
\[ \Rightarrow x = \frac{-1000 - 1000}{2} = -1000 \]

16. \(x^2 + 6x + 9 = 0\)

This equation is in the form of a special quadratic, namely a perfect square trinomial:

\[ x^2 + 6x + 9 = (x + 3)^2 \]

Solve this by taking square roots:

\[ \sqrt{(x + 3)^2} = \sqrt{0} \]
\[ x + 3 = 0 \Rightarrow x = -3 \]

18. \(-4x^2 + 4x = 9\)

Write the equation in standard form; multiply by -1.

\[ 4x^2 - 4x + 9 = 0 \]

Quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Plug in the values \(a = 4, b = -4, c = 9\)

\[ x = \frac{-4 \pm \sqrt{(-4)^2 - 4(4)(9)}}{2(4)} \]

Simplify:

\[ x = \frac{-4 \pm \sqrt{16 - 144}}{8} \]
\[ x = \frac{-4 \pm \sqrt{-128}}{8} \]
Because the discriminant is negative, this equation has no solutions in the real numbers.

20. A moment’s thought shows that the equation can be factored over the integers:

\[x^2 - 2x - 3 = 0\]
\[(x - 3)(x + 1) = 0\]
\[(x - 3) = 0 \Rightarrow x = 3\]
\[(x + 1) = 0 \Rightarrow x = -1\]

Answer: \(x = 3\) or \(x = -1\)

22. Define: Let \(x\) = the smaller odd integer
\(x + 2\) = the next odd integer

Translate: The product of the two numbers is 1 less than 3 times their sum:

\[3(x + x + 2) - 1\]

We can write the equation:

\[x(x + 2) = 3(x + x + 2) - 1\]

Solve:

\[x(x + 2) = 3(x + x + 2) - 1\]
\[x^2 + 2x = 3x + 3x + 6 - 1\]
\[x^2 - 4x - 5 = 0\]

A moment’s reflection shows that the quadratic expression can be factored over the integers:

\[(x - 5)(x + 1) = 0\]
\[(x - 5) = 0 \Rightarrow x = 5\]
\[(x + 1) = 0 \Rightarrow x = -1\]

Since we are looking for positive integers take, \(x = 5\).

Answer: 5 and 7

Check: The answer checks out.

24. Draw a sketch.

Define: Let \(x\) = the length of the side of the square.

Translate: The area of the rectangular piece of plywood is 4.8 = 32 feet squared.

The area of the square corner to be cut off is: \(x \cdot x = x^2\)

The area of the cut off part is one-third of the total area:

\[x^2 = \frac{1}{3}(32)\]

Solve:
\[ \sqrt{x^2} = \sqrt{\frac{32}{3}} \]
\[ x = \pm \sqrt{\frac{32}{3}} \]
\[ x = \sqrt{\frac{32}{3}} \approx 3.27 \]

The length of the square should be approximately 3.27 feet

**Check**: The answers checks out.

**The Discriminant**

2. Rewrite the equation in standard form:

\[ x^2 - 5x - 8 = 0 \]

Plug \( a = 1, b = -5 \) and \( c = -8 \) into the discriminant formula:

\[ D = b^2 - 4ac \]
\[ D = (-5)^2 - 4(1)(-8) \]
\[ D = 25 + 32 \]
\[ D = 57 \]

There are two real solutions because \( D > 0 \).

4. The equation is already in standard form:

\[ x^2 + 3x + 2 = 0 \]

Plug \( a = 1, b = 3 \) and \( c = 2 \) into the discriminant formula:

\[ D = b^2 - 4ac \]
\[ D = (3)^2 - 4(1)(2) \]
\[ D = 9 - 8 \]
\[ D = 1 \]

There are two real solutions because \( D > 0 \).

6. The equation is already in standard form:

\[ -5x^2 + 5x - 6 = 0 \]

Plug \( a = -5, b = 5 \) and \( c = -6 \) into the discriminant formula:

\[ D = b^2 - 4ac \]
\[ D = (5)^2 - 4(-5)(-6) \]
\[ D = 25 - 120 \]
\[ D = -95 \]
There are no real solutions because $D < 0$.

8. Rewrite the equation in standard form:

$$5x^2 - 6x = 0$$

Plug $a = 5, b = -6$ and $c = 0$ into the discriminant formula:

$$D = b^2 - 4ac$$
$$D = (-6)^2 - 4(5)(0)$$
$$D = 36$$

Since $D > 0$, there will be two real solutions. Since the discriminant is a perfect square, these two, distinct real solutions will also be rational.

10. The equation is already in standard form:

$$x^2 - 8x + 16 = 0$$

Plug $a = 1, b = -8$ and $c = 16$ into the discriminant formula:

$$D = b^2 - 4ac$$
$$D = (-8)^2 - 4(1)(16)$$
$$D = 64 - 64$$
$$D = 0$$

There is one real solution because $D = 0$.

12. The equation is already in standard form:

$$x^2 - 64 = 0$$

Plug $a = 1, b = 0$ and $c = -64$ into the discriminant formula:

$$D = b^2 - 4ac$$
$$D = (0)^2 - 4(1)(-64)$$
$$D = 256$$

Since $D > 0$, there will be two real solutions. Since the discriminant is a perfect square, these two, distinct real solutions will also be rational.

14. The equation is already in standard form:

$$x^2 + 2x - 3 = 0$$

Plug $a = 1, b = 2$ and $c = -3$ into the discriminant formula:

$$D = b^2 - 4ac$$
$$D = (2)^2 - 4(1)(-3)$$
$$D = 4 + 12$$
$$D = 16$$
Since $D > 0$, there will be two real solutions. Since the discriminant is a perfect square, these two, distinct real solutions will also be rational.

16. The equation is already in standard form:

$$\frac{1}{2}x^2 + 2x + \frac{2}{3} = 0$$

Plug $a = \frac{1}{2}$, $b = 2$ and $c = \frac{2}{3}$ into the discriminant formula:

$$D = b^2 - 4ac$$

$$D = (2)^2 - 4 \left( \frac{1}{2} \right) \left( \frac{2}{3} \right)$$

$$D = 4 - \frac{4}{3}$$

$$D = \frac{8}{3}$$

Since the discriminant is not a perfect square, the two, distinct solutions will be irrational.

18. Rewrite the equation in standard form:

$$x^2 - 5x = 0$$

Plug $a = 1$, $b = -5$ and $c = 0$ into the discriminant formula:

$$D = b^2 - 4ac$$

$$D = (-5)^2 - 4(1)(0)$$

$$D = 25$$

Since $D > 0$, there will be two real solutions. Since the discriminant is a perfect square, these two, distinct real solutions will also be rational.

20. Set $R = 20,000$

$$x(200 - 0.4x) = 20000$$

$$200x - 0.4x^2 = 20000$$

$$-0.4x^2 + 200x - 20000 = 0$$

Plug $a = -0.4$, $b = 200$ and $c = -20000$ into the discriminant formula:

$$D = b^2 - 4ac$$

$$D = (200)^2 - 4(-0.4)(-20000)$$

$$D = 40000 - 32000$$

$$D = 8000$$

Since the discriminant is positive, we know that it is possible for Bryson’s business to generate $20,000 in the month of July.
Linear, Exponential and Quadratic Models

2. If we take the difference between consecutive $y$–values, we see that each time the $x$–value increases by one, the $y$–value does not remain constant. Since the difference is not the same, the function is not linear.

4. When we increase the $x$–value by one, the value of $y$ increases by different values. However, the increase is constant: the difference of the difference is always 5 when we increase the $x$–value by one. The function describing these set of values is quadratic.

6. When we increase the $x$–value by one, the value of $y$ increases by different values. Checking the difference of the differences is not constant when we increase the $x$–value by one. Therefore, the function describing these set of values is not quadratic.

8. If we take the ratio of consecutive $y$–values, we see that each time the $x$–value increases by one, the $y$–value is multiplied by 1.5. Since the ratio is always the same, the function is exponential.

10. If we take the ratio of consecutive $y$–values, we see that each time the $x$–value increases by one, the $y$–value is multiplied by 1.25. Since the ratio is always the same, the function is exponential.

To find the equation for the function that represents these values, we start with the general form of an exponential function: $y = a \cdot b^x$. Where $b$ is the ratio between the values of $y$ each time that $x$ is increased by one. The constant $a$ is the value of the function when $x = 0$. Therefore, the function is: $y = 400(1.25)^x$.

12. When we increase the $x$–value by one, the value of $y$ increases by different values. However, the increase is constant: the difference of the difference is always 10 when we increase the $x$–value by one. The function describing these set of values is quadratic.

To find the equation for the function that represents these values, we start with the general form of a quadratic function: $y = ax^2 + bx + c$. We need to use the values in the table to find the values of the constants: $a$, $b$ and $c$. The value of $c$ represents the value of the function when $x = 0$, so $c = -4$.

Then, $y = ax^2 + bx - 4$

Plug in the point $(1, -2)$:

$$-2 = a(1)^2 + b(1) - 4$$

which simplifies to

$$a + b = 2$$

Plug in the point $(-1, -2)$:

$$-2 = a(-1)^2 + b(-1) - 4$$

which simplifies to

$$a - b = 2$$

To find $a$ and $b$, we solve the system of equations:

$$a + b = 2$$
$$a - b = 2$$

Solve the first equation for $b$: 

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\[ b = 2 - a \]

Plug the first equation into the second:
\[ a - (2 - a) = 2 \]

Solve for \( a \):
\[
\begin{align*}
  a - 2 + a &= 2 \\
 2a &= 4 \\
  a &= 2
\end{align*}
\]

Back-substitute:
\[
\begin{align*}
  b &= 2 - 2 \\
  b &= 0
\end{align*}
\]

Therefore the equation of the quadratic function is:
\[
y = 2x^2 - 4
\]

14a.

14b. Assuming the constant term is zero, the best-fit exponential function is \( y = 253.4(0.7967)^x \) (note SE gives the slightly incorrect function \( y = 255.25(0.79)^x \))

14c.
Using the best-fit function, the amount of material remaining after 10 days is:

\[ y = 253.4(0.7967)^x \text{ grams} \]  

\textit{(note SE gives answer 24.17 grams)}

**Problem Solving Strategies: Choose a Function Model**

2a. In Example 1, the linear function \( y = 0.2x + 2 \) was obtained. To find the length of the spring when the weight is 15 ounces, we plug in \( x = 15 \):

\[ y = 0.2(15) + 2 = 5 \text{ inches} \]

2b. To find the length of the spring when the weight is 15 ounces using the cubic function in Example 1, we plug in \( x = 15 \):

\[
y = -0.000145x^3 - 0.000221x^2 + 0.202x + 2.002 \\
y = -0.000145(15)^3 - 0.000221(15)^2 + 0.202(15) + 2.002 \approx 4.5 \text{ inches}
\]

2c. Examining the graphs of the linear and the cubic functions in Example 1, we see that the cubic function gives a better approximation to the actual shape of the data points. For a smaller weight (below about 8 oz), either approximation would work well.

4a. In Example 2, the linear function \( y = -3.58x + 50 \) was obtained. To find the height of the water after 19 seconds, we plug in \( x = 19 \):

\[ y = -3.58(19) + 50 = -18.02 \text{ inches} \]

4b. To find the height of the water after 19 seconds using the quadratic function in Example 2, we plug in \( x = 19 \):

\[
y = 0.075x^2 - 3.87x + 50 \\
y = 0.075(19)^2 - 3.87(19) + 50 \approx 3.55 \text{ inches}
\]

4c. The quadratic function clearly gives a better answer because the height obtained from the linear function was negative, which is impossible \textit{(note SE’s answer of “more accurate” is quite vague in this context, especially when one of the functions gives a negative answer)}

6a. In Example 4, the linear function \( y = 2(1.5)^x \) was obtained \textit{(note SE has a typo here: 2(1.5)x)}. To find the number of fish in generation 8, we plug in \( x = 8 \):

\[ y = 2(1.5)^8 \approx 51,000 \text{ fish} \]

6b. To find the number of fish in generation 8 using the logistic function in Example 4, we plug in \( x = 8 \):

\[
y = \frac{2023.6}{1 + 1706.3(2.71)^{-0.484x}} \\
y = \frac{2023.6}{1 + 1706.3(2.71)^{-0.484(8)}} \approx 55,000 \text{ fish.}
\]

6c. The answers are very close in both cases, since the number of generations is relatively small. The exponential function \textit{(note SE mistakenly says linear function here)} would be best to use here because of its simpler form. For larger numbers of generations (greater than about 15), though, the logistic function is a much better fit to the data.
Chapter 11

TE Algebra and Geometry Connections; Working with Data - Solution Key

11.1 Complete Solutions to Even-Numbered Review Questions

Graphs of Square Root Functions
4. 

6. Note that we can write $y = \sqrt{4x + 4}$ as $y = \sqrt{4(x + 1)} = 2\sqrt{x+1}$. So this graph is obtained from the graph of $y = \sqrt{x}$ by stretching by a factor of 2, and shifting 1 unit to the left.

8. $y = 2\sqrt{x} + 5$ is obtained from the graph of $y = \sqrt{x}$ by stretching by a factor of 2, and shifting 5 units upward.

10. $y = 4 + 2\sqrt{x}$ is obtained from the graph of $y = \sqrt{x}$ by stretching by a factor of 2, and shifting 4 units upward.
14. The equation required here is: \( T = a \sqrt{L} \) where \( a = \frac{2\pi}{\sqrt{32}} \). If \( T = 2 \), then substituting all these into the first results in:

\[
2 = \frac{2\pi}{\sqrt{32}} \sqrt{L}
\]

Multiply both sides by the reciprocal of the fraction:

\[
\frac{\sqrt{32}}{2\pi} \cdot 2 = \frac{\sqrt{32}}{2\pi} \cdot \frac{2\pi}{\sqrt{32}} \sqrt{L}
\]

Simplify:

\[
\frac{\sqrt{32}}{\pi} = \sqrt{L}
\]

To isolate \( L \), square both sides:

\[
\left( \frac{\sqrt{32}}{\pi} \right)^2 = (\sqrt{L})^2
\]

\[
\frac{32}{\pi^2} = L
\]

\( L \approx 3.24 \) feet

Note that the same approximated answer could have been obtained from the graph of:

\[
T = \frac{2\pi}{\sqrt{32}} \sqrt{L}
\]

or written with \( x \) and \( y \):

\[
y = \frac{\pi}{2 \sqrt{2}} \sqrt{x}
\]

16. The equation required here is: \( T = a \sqrt{L} \) where \( a = \frac{2\pi}{\sqrt{3.69}} \). If \( T = 3 \), then substituting all these into the first results in:

\[
3 = \frac{2\pi}{\sqrt{3.69}} \sqrt{L}
\]

Multiply both sides by the reciprocal of the fraction:

\[
\frac{\sqrt{3.69}}{2\pi} \cdot 3 = \frac{\sqrt{3.69}}{2\pi} \cdot \frac{2\pi}{\sqrt{3.69}} \sqrt{L}
\]
Simplify:

\[ \frac{3 \sqrt{3.69}}{2\pi} = \sqrt{L} \]

To isolate \( L \), square both sides:

\[ \left( \frac{3 \sqrt{3.69}}{2\pi} \right)^2 = (\sqrt{L})^2 \]

\[ \frac{9(3.69)}{4\pi^2} = L \]

\[ L \approx 0.84 \text{ meters} \]

Note that the same approximated answer could have been obtained from the graph of:

\[ T = \frac{2\pi}{\sqrt{3.69}} \sqrt{L} \]

or written with \( x \) and \( y \):

\[ y = \frac{2\pi}{\sqrt{3.69}} \sqrt{x} \]

18. Let \( d \) = length of the diagonal and \( x \) = horizontal length. Then \( 2.39 \text{(vertical length)} = 1 \times \text{horizontal length} \). Or, vertical length \( = \left( \frac{1}{2.39} \right)x \). The area of the screen is: \( A = \text{(length)}\text{(width)} \) or \( A = \left( \frac{1}{2.39} \right)x^2 \).

Find how the diagonal length and the horizontal length are related by using the Pythagorean theorem:

\[ d^2 = x^2 + \left( \frac{1}{2.39} \right)^2 x^2 \]

which simplifies to:

\[ d^2 = x^2 + \left( \frac{1}{2.39} \right)^2 x^2 \]

\[ d^2 = \left( 1 + \frac{1}{(2.39)^2} \right)x^2 \]

\[ d^2 = \left( \frac{6.7121}{5.7121} \right)x^2 \]

Solve for \( x^2 \):

\[ \left( \frac{5.7121}{6.7121} \right)d^2 = x^2 \]
Substitute in the area formula:

\[ A = \left( \frac{1}{2.39} \right) x^2 \]

\[ A = \left( \frac{1}{2.39} \right) \left( \frac{5.7121}{6.7121} \right) d^2 \]

Simplify:

\[ A = \left( \frac{2.39}{6.7121} \right) d^2 \]

Solve for \( d \):

\[ \sqrt{\left( \frac{6.7121}{2.39} \right) A} = d \]

\[ \sqrt{\left( \frac{6.7121}{2.39} \right) A} = \sqrt{d^2} \]

\[ \sqrt{\left( \frac{6.7121}{2.39} \right) A} = d \]

\[ \sqrt{\left( \frac{1}{2.39} + 2.39 \right) \sqrt{A}} = d \]

which represents the diagonal length as a function of the area.

Make a graph where the horizontal axis represents the area of the television screen and the vertical axis is the length of the diagonal. From the graph we can estimate that when the area of a TV screen is 150 inches squared, the length of the diagonal is approximately 20.5 inches. We can confirm this by plugging in \( A = 150 \) into the formula that relates the diagonal to the area.

20. Window: \(-5 \leq x \leq 5; \quad 0 \leq y \leq 10\)

![Graph 1](image1.png)

22. Window: \(0 \leq x \leq 5; \quad -3 \leq y \leq 1\)

![Graph 2](image2.png)
Radical Expressions

2. \(\sqrt{81} = 3\) since \(3 \cdot 3 \cdot 3 = 81\)

4. \(\sqrt[4]{1024} = 4\) since \(4 \cdot 4 \cdot 4 \cdot 4 = 1024\)

6. \(\sqrt[4]{w} = z^{1/4}w^{1/4}\)

8. \(\sqrt[3]{y} = y^{1/3}\)

10. \(\sqrt[3]{300} = \sqrt[3]{3 \cdot 100} = \sqrt[3]{3 \cdot 10^2} = 10\sqrt[3]{3}\)

12. \(\sqrt[3]{\frac{240}{507}} = \sqrt[3]{\frac{16 \cdot 15}{81 \cdot 7}} = \sqrt[3]{\frac{16}{81}} \cdot \sqrt[3]{\frac{15}{7}} = \sqrt[3]{\frac{16}{81}} \cdot \sqrt[3]{\frac{15}{7}} = \frac{4}{9} \cdot \sqrt[3]{\frac{15}{7}}\)

14. \(\sqrt[3]{64x^8} = \sqrt[3]{26x^6} \cdot x^2 = 2x \sqrt[3]{x^2}\)

16. \(\frac{3}{10 \sqrt[3]{y^4}} = \frac{3}{10} \cdot \frac{\sqrt[3]{8 \cdot 2 \cdot x^2 \cdot x^2}}{\sqrt[3]{27 \cdot 5 \cdot y^3 \cdot y}} = \frac{3}{\sqrt[3]{8 \cdot 27 \cdot 5 \cdot x^2 \cdot y}} \cdot \frac{\sqrt[3]{x^2 \cdot y}}{\sqrt[3]{x^2 \cdot y}} = \frac{3}{\sqrt[3]{x^2 \cdot y}} \cdot \frac{\sqrt[3]{x^2 \cdot y}}{\sqrt[3]{x^2 \cdot y}} = \frac{3 \cdot x^2}{3 \cdot x^2} \cdot \frac{\sqrt[3]{x^2 \cdot y}}{\sqrt[3]{x^2 \cdot y}}\)

18. \(\sqrt[3]{80 + 6 \sqrt[3]{405}} = 3 \cdot 5 + 6 \sqrt[3]{\sqrt[3]{81} \cdot 5} = 3 \cdot 5 + 6 \cdot 5 = 15 + 30 = 45\)

20. \(\sqrt[3]{8x^3 - 4x \sqrt[3]{98x}} = \sqrt[3]{2^2 \cdot 2 \cdot x^2 \cdot x - 4x \sqrt[3]{81} \cdot 2x} = 2x \sqrt[3]{2x} - 28x \sqrt[3]{2x}\)

22. \(\sqrt[3]{x^3} + \sqrt[3]{y^3} = \sqrt[3]{x^3 \cdot 4} + \sqrt[3]{y^3 \cdot 4} = x \sqrt[3]{x} + 4x \sqrt[3]{4} = 5x \sqrt[3]{4}\)

24. This is the product of a sum and difference of two terms; it expands into a difference of squares:

\[(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a} + \sqrt[3]{b}) = (\sqrt[3]{a})^2 - (\sqrt[3]{b})^2 = a - b\]

26. \(\frac{7}{\sqrt[15]{9}} = \frac{7}{\sqrt[15]{9}} \cdot \frac{\sqrt[15]{9}}{\sqrt[15]{9}} = \frac{7 \sqrt[15]{9}}{15}\)

28. \(\frac{2x}{\sqrt[5]{x}} = \frac{2x}{\sqrt[5]{x}} \cdot \frac{\sqrt[5]{x}}{\sqrt[5]{x}} = 2 \sqrt[5]{x^2} \cdot \sqrt[5]{x} = 2 \sqrt[5]{x^3}\)

30. \(\frac{12}{2 \sqrt[5]{15}} = \frac{12}{2 \sqrt[5]{15}} \cdot \frac{2 \sqrt[5]{15}}{2 \sqrt[5]{15}} = \frac{12(2 \sqrt[5]{15})}{2^2(\sqrt[5]{15})^2} = -24 - 12 \sqrt[5]{15}\)

32. \(\frac{x}{\sqrt[2]{x} + \sqrt[5]{x}} = \frac{x}{\sqrt[2]{x} + \sqrt[5]{x}} \cdot \frac{\sqrt[2]{x} - \sqrt[5]{x}}{\sqrt[2]{x} - \sqrt[5]{x}} = \frac{x \sqrt[2]{x} - x \sqrt[5]{x}}{(\sqrt[2]{x})^2 - (\sqrt[5]{x})^2} = \frac{x \sqrt[2]{x} - x \sqrt[5]{x}}{2 - x}\)

34. Given that \(V = \frac{4}{3} \pi R^3\), set \(R = 950\) and solve:

\[950 = \frac{4}{3} \pi R^3\]
\[\frac{3}{4} \cdot 950 = \pi R^3\]
\[\frac{3 \cdot 475}{2\pi} = R^3\]
\[\frac{3 \cdot 475}{2\pi} = \frac{3}{2\pi} \cdot R^3\]

\[R = \sqrt[3]{\frac{3 \cdot 475}{2\pi}} \approx 6.1\ cm\]

Radical Equations

2. \(\sqrt{3x - 1} = 5\)

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Square each side of the equation:

\[(\sqrt{3x - 1})^2 = 5^2\]

Simplify:

\[3x - 1 = 25\]

Solve for \(x\):

\[x = 26/3\]

The answer checks out.

4. Isolate the radical on one side of the equation:

\[\sqrt[3]{x - 3} = 1\]

Raise each side of the equation to the third power:

\[(\sqrt[3]{x - 3})^3 = 1^3\]

Simplify:

\[x - 3 = 1\]

Solve for \(x\):

\[x = 4\]

The answer checks out.

6. Isolate the radical on one side of the equation:

\[\sqrt[3]{-2 - 5x} = -3\]

Raise each side of the equation to the third power:

\[(\sqrt[3]{-2 - 5x})^3 = (-3)^3\]

Simplify:

\[-2 - 5x = -27\]

Solve for \(x\):

\[x = 5\]

The answer checks out.

8. Isolate the radical on one side of the equation:
\[ \sqrt{x^2 - 5x - 6} = 0 \]
\[ \sqrt{x^2 - 5x} = 6 \]

Square each side of the equation:

\[ (\sqrt{x^2 - 5x})^2 = 6^2 \]

Simplify:

\[ x^2 - 5x = 36 \]

Solve for \( x \) by factoring the quadratic equation:

\[ x^2 - 5x - 36 = 0 \]
\[ (x - 9)(x + 4) = 0 \]
\[ (x - 9) = 0 \Rightarrow x = 9 \]
\[ (x + 4) = 0 \Rightarrow x = -4 \]

The answers check out.

10. Isolate the radical on one side of the equation:

\[ \sqrt{x + 6} = x + 4 \]

Square each side of the equation:

\[ (\sqrt{x + 6})^2 = (x + 4)^2 \]

Simplify:

\[ x + 6 = x^2 + 8x + 16 \]

Solve for \( x \) by factoring the quadratic equation:

\[ x^2 + 7x + 10 = 0 \]
\[ (x + 5)(x + 2) = 0 \]
\[ (x + 5) = 0 \Rightarrow x = -5 \]
\[ (x + 2) = 0 \Rightarrow x = -2 \]

The answer \( x = -2 \) checks out (\(-5\) is an extraneous solution).

12. The square root of any number is never negative. Therefore, this equation has no real solutions.

14. Isolate one of the radicals on one side of the equation:

\[ \sqrt{2x - 2} = 2\sqrt{x} - 2 \]

Square each side of the equation:
\[
(\sqrt{2x-2})^2 = (2\sqrt{x} - 2)^2
\]

Simplify:

\[
2x - 2 = (2\sqrt{x} - 2)(2\sqrt{x} - 2) \\
2x - 2 = 4(\sqrt{x})^2 - 4\sqrt{x} - 4\sqrt{x} + 4 \\
2x - 2 = 4x - 8\sqrt{x} + 4 \\
8\sqrt{x} = 2x + 6
\]

Divide both sides by 2:

\[
4\sqrt{x} = x + 3
\]

Square each side of the equation:

\[
(4\sqrt{x})^2 = (x + 3)^2 \\
16x = x^2 + 6x + 9 \\
x^2 - 10x + 9 = 0
\]

Solve for \(x\) by factoring the quadratic equation:

\[
(x - 9)(x - 1) = 0 \\
(x - 9) = 0 \Rightarrow x = 9 \\
(x - 1) = 0 \Rightarrow x = 1
\]

The answers check out.

16. Since the more complicated radical is already isolated on one side of the equation, just square both sides:

\[
(3\sqrt{x} - 9)^2 = (\sqrt{2x - 14})^2 \\
9x - 54\sqrt{x} + 81 = 2x - 14
\]

Simplify and isolate the remaining radical:

\[
7x + 95 = 54\sqrt{x}
\]

Square both sides:

\[
49x^2 + 1330x + 9025 = 2916x
\]

Put the quadratic in standard form:

\[
49x^2 - 1586x + 9025 = 0
\]

Solve using the quadratic formula:
\[ x = \frac{1586 \pm \sqrt{1586^2 - 4(49)(9025)}}{2(49)} \]
\[ x = \frac{1586 \pm 864}{98} \]
\[ \Rightarrow x = 25, x = 361/49 \]

The solution \( x = 25 \) checks out, but the solution \( x = 361/49 \) is extraneous.

18. Plugging 124 into the area formula, \( 124 = \pi r^2 \), so \( r = \sqrt{124/\pi} \). Plug this into the circumference formula: \( C = 2\pi\sqrt{124/\pi} \approx 39.47 \text{ inches} \) (note SE says 39.46 inches)

20. Replace \( h \) with 120 in the equation:

\[ h = -16t^2 + 256 \]
\[ 120 = -16t^2 + 256 \]

Solve the quadratic equation by square roots:

\[ 16t^2 = 256 - 120 \]
\[ 16t^2 = 136 \]
\[ t^2 = \frac{136}{16} = \frac{17}{2} \]
\[ \sqrt{t^2} = \sqrt{\frac{17}{2}} \]
\[ t = \sqrt{\frac{17}{2}} \approx 2.9 \text{ seconds} \]

The Pythagorean Theorem and Its Converse

2. Since \( 6\sqrt{2} > 6 \), the length of the hypotenuse must be \( 6\sqrt{2} \). Using the Pythagorean Theorem to verify:

\[ ? \]
\[ (6)^2 + (6)^2 = (6\sqrt{2})^2 \]
\[ ? \]
\[ 36 + 36 = 36.2 \]
\[ 72 = 72 \]

Since the triple of numbers satisfy the Pythagorean Theorem, they form the side lengths of a right triangle.

4.

\[ a^2 + b^2 = c^2 \]
\[ 12^2 + 16^2 = c^2 \]
\[ 400 = c^2 \]
\[ c = 20 \]

6.
\[ a^2 + b^2 = c^2 \]
\[ 4^2 + b^2 = 11^2 \]
\[ 16 + b^2 = 121 \]
\[ b^2 = 121 - 16 \]
\[ b^2 = 105 \]
\[ \sqrt{b^2} = \sqrt{105} \]
\[ b = \sqrt{105} \]

8.
\[ a^2 + 21^2 = 35^2 \]
\[ a^2 = 35^2 - 21^2 = 1225 - 441 = 784 \]
\[ a = 28 \]

10. Let \( x \) = length of the hypotenuse. Then the other side measures: \( x - 4 \).

\[
(x - 4)^2 + 12^2 = x^2 \\
x^2 - 8x + 16 + 144 = x^2 \\
8x = 160 \\
x = 20
\]

Answer: The lengths of the sides of the triangle are: 20, 16, 12.

12. To go from second base to home plate, we traverse a diagonal of the square. Let the diagonal have length \( c \). The diagonal is the hypotenuse of a right triangle with both legs equal to the length of the side of the square, so we can use the Pythagorean Theorem:

\[
90^2 + 90^2 = c^2 \\
\sqrt{2 \cdot 90^2} = c \\
90 \sqrt{2} = c, \text{ so the distance is } 90 \sqrt{2} \approx 127.3 \text{ feet.}
\]

14. The height of the ladder will be a leg of a right triangle with hypotenuse 10 feet (the length of the ladder) and other leg 6 feet (the distance from the house). Let the height of the ladder be \( h \). We can use the Pythagorean Theorem:

\[
6^2 + h^2 = 10^2 \\
h^2 = 100 - 36 = 64 \\
h = 8, \text{ so the ladder touches the house 8 feet above the ground.}
\]

16. Let \( x \) = length of the long side of the field and \( d \) the length of the hypotenuse. Then an expression for the hypotenuse can be found using the Pythagorean Theorem:

\[
d^2 = x^2 + 123^2 \\
\sqrt{d^2} = \sqrt{x^2 + 123^2} \\
d = \sqrt{x^2 + 123^2}
\]
By “walking the hypotenuse”, Marcus saves a distance equal to half of the long side of the field. This means:

distance walking along the two sides = \(x + 123\)

distance saved by “walking the hypotenuse” = sum of the two legs – length of the hypotenuse = \((x + 123) - \sqrt{x^2 + 123^2}\)

Putting it all together:

\[(x + 123) - \sqrt{x^2 + 123^2} = \frac{x}{2}\]

Solve for \(x\):

\[\sqrt{x^2 + 123^2} = \frac{x}{2} + x + 123\]

\[\sqrt{x^2 + 123^2} = \frac{x}{2} + 123\]

\[(\sqrt{x^2 + 123^2})^2 = \left(\frac{x}{2} + 123\right)^2\]

\[x^2 + 123^2 = \left(\frac{x}{2}\right)^2 + 2 \cdot \left(\frac{x}{2}\right) \cdot 123 + 123^2\]

\[x^2 + 123^2 = \frac{x^2}{4} + 123x + 123^2\]

\[\frac{3}{4}x^2 - 123x = 0\]

\[3x^2 - 492x = 0\]

\[x(3x - 492) = 0\]

\[x = 0\]

\[3x - 492) = 0 \Rightarrow x = \frac{492}{3} = 164\]

Answer: The long side of the field measures 164 feet.

18. The area of a circle is \(A = \pi r^2\) where \(r\) is the radius of the circle. By the Pythagorean Theorem, the diagonal \(d\) can be found:

\[5^2 + 9^2 = d^2\]

\[25 + 81 = d^2\]

\[106 = d^2\]

\[\sqrt{106} = d\]

The radius is half the diameter:

\[r = \frac{\sqrt{106}}{2}\]

The area can now be calculated:

\[A = \pi r^2 = r \left(\frac{\sqrt{106}}{2}\right)^2\]

\[A = \frac{106}{4} \pi = \frac{53}{2} \pi \approx 83\text{ ft. sq.}\]
Distance and Midpoint Formulas

2. Plug the values of the two points into the distance formula:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(-1 - 4)^2 + (0 - 2)^2} \]
\[ d = \sqrt{(-5)^2 + (-2)^2} \]
\[ d = \sqrt{25 + 4} \]
\[ d = \sqrt{29} \]

4. Plug the values of the two points into the distance formula:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(0.5 - 4)^2 + (-2.5 - (-4))^2} \]
\[ d = \sqrt{(-3.5)^2 + (1.5)^2} \]
\[ d = \sqrt{12.25 + 2.25} \]
\[ d = \sqrt{14.5} \approx 3.81 \]

6. Plug the values of the two points into the distance formula:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(-3.4 - 2.3)^2 + (-5.2 - 4.5)^2} \]
\[ d = \sqrt{(-5.7)^2 + (-9.7)^2} \]
\[ d = \sqrt{32.49 + 94.09} \]
\[ d = \sqrt{126.58} \approx 11.25 \]

8. We advise making a sketch of the given situation: Draw line segments from point \((-2, 5)\) to the line \(y = 3\). Let \(k\) be the missing value of \(x\) we are seeking. Let’s use the distance formula.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ 8 = \sqrt{(-2 - k)^2 + (5 - 3)^2} \]

Square both sides of the equation:

\[ 8^2 = \left( \sqrt{(-2 - k)^2 + (5 - 3)^2} \right)^2 \]
\[ 64 = (-2 - k)^2 + (5 - 3)^2 \]
\[ 64 = 4 + 4k + k^2 + 4 \]
\[ k^2 + 4k - 56 = 0 \]

Use the quadratic formula:
\[
k = \frac{-4 \pm \sqrt{4^2 - 4(1)(-56)}}{2(1)}
\]
\[
\Rightarrow k = \frac{-4 + \sqrt{16 \cdot 15}}{2} = \frac{-4 + 4\sqrt{15}}{2} = -2 + 2\sqrt{15}
\]
\[
\Rightarrow k = \frac{-4 - \sqrt{16 \cdot 15}}{2} = \frac{-4 - 4\sqrt{15}}{2} = -2 - 2\sqrt{15}
\]

Answer: The points are \((-2 + 2\sqrt{15}, 3)\) and \((-2 - 2\sqrt{15}, 3)\)

The midpoint of the line segment joining the two points is:
\[
\left(\frac{(-2 + 2\sqrt{15}) + (-2 - 2\sqrt{15})}{2}, \frac{3 + 3}{2}\right)
\]
\[
\left(\frac{-4}{2}, \frac{6}{2}\right)
\]
\[
(-2, 3)
\]

10. Plug the values of the two points into the midpoint formula:
\[
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
\[
M = \left(2 + 2, -3 + 4\right)
\]
\[
M = \left(4, \frac{1}{2}\right)
\]

12. Plug the values of the two points into the midpoint formula:
\[
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
\[
M = \left(\frac{1.8 + (-4.0)}{2}, \frac{-3 + 1.4}{2}\right)
\]
\[
M = (0.7, -1)
\]

14. Plug the values of the two points into the midpoint formula:
\[
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
\[
M = \left(\frac{10 + 2}{2}, \frac{2 + (-4)}{2}\right)
\]
\[
M = (6, -1)
\]

16. Let the other endpoint be denoted as \((k, l)\). Then the midpoint formula gives:
\[
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
\]
\[
(0, 4) = \left(\frac{-10 + k}{2}, \frac{-2 + l}{2}\right)
\]

Each coordinate yields an equation:
\[ \Rightarrow \frac{-10 + k}{2} = 0 \Rightarrow k = 0 \]
\[ \Rightarrow \frac{-2 + l}{2} = 4 \Rightarrow l = 10 \]

Therefore, the other endpoint is: \((10, 10)\).

18. We can find the length of the sides using the distance formula. For example, the length of \(AB\) is:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
d = \sqrt{(4 - 3)^2 + (3 - 4)^2}
\]
\[
d = \sqrt{1^2 + (-7)^2}
\]
\[
d = \sqrt{50} \approx 7.07
\]

In an identical manner, one can show that all the other sides have length equal to \(\sqrt{50}\).

20. The area of a circle is \(A = \pi r^2\) where \(r\) is the radius of the circle. The radius can be calculated using the distance formula:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
\[
r = \sqrt{(-5 - 3)^2 + (4 - 2)^2}
\]
\[
r = \sqrt{(-8)^2 + (2)^2}
\]
\[
r = \sqrt{64 + 4}
\]
\[
r = \sqrt{68}
\]
\[
r = \sqrt{4.17}
\]
\[
r = 2 \sqrt{17}
\]

Therefore, the area of the circle is:

\[
A = \pi r^2
\]
\[
A = \pi (2 \sqrt{17})^2
\]
\[
A = 68\pi
\]
Measures of Central Tendency and Dispersion

2. The mean is calculated using the formula:

\[
\overline{x} = \frac{\sum x_i}{n}
\]

The mean, \(\overline{x}\), is found by adding up the first column and dividing by 20, the total number of data points. The mean is 21.75.

The median is found by taking the middle two data points in this ordered list (column 1) and finding their average. The average of the 10th and 11th data points is: \(\frac{15+15}{2} = 15\).

The standard deviation, \(s\), can be found by the following steps:

1. To find the entries in column 2, subtract each data point from the mean (already found).
2. To find the entries in the third column, square each corresponding entry in the second column obtained in the previous step.
3. Add up each entry in the third column and divide by 20.
4. Take the square root of the number obtained in the previous step.

The standard deviation of the data set is: 33.065

Because of the outlier, the median does a better job of summarizing the central tendency in the data set.

4a. The median would be most appropriate here. Goldfish lifetimes have a huge variance: some may live for weeks, while some may die in a matter of days. This variance is not reflected in the mean.

4b. The mode would be appropriate here, because the presence of outliers (perhaps older siblings or parents watching TV with their children) would skew both the mean and the median.
4c. The mean would be appropriate here, because the weights of bags marked as 5 lbs are likely clustered fairly closely about the central value of 5 lbs.

**Stem-and-Leaf Plots and Histograms**

2.

<table>
<thead>
<tr>
<th>Bin Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 ≤ x &lt; 650</td>
<td>1</td>
</tr>
<tr>
<td>650 ≤ x &lt; 700</td>
<td>1</td>
</tr>
<tr>
<td>700 ≤ x &lt; 750</td>
<td>3</td>
</tr>
<tr>
<td>750 ≤ x &lt; 800</td>
<td>6</td>
</tr>
<tr>
<td>800 ≤ x &lt; 850</td>
<td>8</td>
</tr>
<tr>
<td>850 ≤ x &lt; 900</td>
<td>10</td>
</tr>
<tr>
<td>900 ≤ x &lt; 950</td>
<td>5</td>
</tr>
<tr>
<td>950 ≤ x &lt; 1000</td>
<td>6</td>
</tr>
</tbody>
</table>

4a. The mode can be read off the table immediately: 32 mph, the only number that appears four times. Since the number of data points is even, the median is the average of the 20th and 21st values, or 35.5 mph. The average is easily calculated to be 34.9 mph.

4b.

<table>
<thead>
<tr>
<th>Bin Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 ≤ x &lt; 30</td>
<td>5</td>
</tr>
<tr>
<td>30 ≤ x &lt; 35</td>
<td>12</td>
</tr>
<tr>
<td>35 ≤ x &lt; 40</td>
<td>10</td>
</tr>
<tr>
<td>40 ≤ x &lt; 45</td>
<td>7</td>
</tr>
<tr>
<td>45 ≤ x &lt; 50</td>
<td>4</td>
</tr>
<tr>
<td>50 ≤ x &lt; 55</td>
<td>1</td>
</tr>
<tr>
<td>55 ≤ x &lt; 60</td>
<td>1</td>
</tr>
</tbody>
</table>
Box-and-Whisker Plots

2. The box-and-whisker plot for many runs is shown. It includes values that are less likely to occur (3 and 18) so the range is greater than for a small number of runs. The median is the same and the IQR is similar, indicating that a small number of trials results in a good estimate of these quantities.

4. The box-and-whisker plot is shown. Since points $a$, $c$, and $d$ are within 1.5 times the IQR, the only outlier would be point $b$. 
Chapter 12

TE Rational Equations and Functions; Topics in Statistics - Solution Key

12.1 Complete Solutions to Even-Numbered Review Questions

Inverse Variation Models

\[ y = \frac{10}{x} \]
6. Since $y$ is inversely proportional to $x$, then the general relationship tells us:

$$y = \frac{k}{x}$$

Plug in the values $y = 2$ and $x = 8$:

$$2 = \frac{k}{8}$$

Solve for $k$ by multiplying both sides of the equation by 8:

$$k = 16$$

Now we put $k$ back into the general equation. The inverse relationship is given by:

$$y = \frac{16}{x}$$

When $x = 12$,

$$y = \frac{16}{12} = \frac{4}{3}$$

8. Since $w$ is inversely proportional to the square of $u$, then the general relationship tells us:

$$w = \frac{k}{u^2}$$

Plug in the values $w = 4$ and $u = 2$:

$$4 = \frac{k}{2^2}$$

Solve for $k$ by multiplying both sides of the equation by 4:
\[ k = 16 \]

Now we put \( k \) back into the general equation. The inverse relationship is given by:

\[ w = \frac{16}{u^2} \]

When \( u = 8 \),

\[ w = \frac{16}{8^2} = \frac{1}{4} \]

10. Since \( a \) varies directly with \( b \) and inversely with the square of \( c \), then the general relationship tells us:

\[ a = \frac{b}{c^2} k \]

Plug in the values \( a = 10 \), \( b = 5 \) and \( c = 2 \):

\[ 10 = \frac{5}{2^2} k \]

Solve for \( k \) by multiplying both sides of the equation by \( \frac{4}{5} \):

\[ k = 8 \]

Now we put \( k \) back into the general equation. The inverse relationship is given by:

\[ a = \frac{8b}{c^2} \]

When \( b = 3 \) and \( c = 6 \),

\[ a = \frac{8 \cdot 3}{6^2} = \frac{24}{36} = \frac{2}{3} \]

12. Call the current \( I \) and the resistance \( R \). Since \( I \) is inversely proportional to \( R \), then the general relationship tells us:

\[ IR = k \]

Plug in the values \( I = 2.5 \) and \( R = 20 \):

\[ (2.5)(20) = k \]

\[ k = 50 \]

Now we put \( k \) back into the general equation. The inverse relationship is given by:

\[ IR = 50 \]

When \( I = 5 \),
(5) \( R = 50 \)
\[ R = 10 \text{ ohms.} \]

14. Since \( V \) varies jointly as the height, \( h \), and the square of the length of the base, \( l \), then the general relationship tells us:

\[ V = k \cdot h \cdot l^2 \]

Plug in the values \( h = 4 \), \( l = 3 \) and \( V = 12 \):

\[ 12 = k \cdot 4 \cdot 3^2 \]

Solve for \( k \) by multiplying both sides of the equation by 36:

\[ k = \frac{1}{3} \]

Now we put \( k \) back into the general equation. The inverse relationship is given by:

\[ V = \frac{1}{3} \cdot h \cdot l^2 \]

When \( h = 9 \) and \( l = 5 \),

\[ V = \frac{1}{3} \cdot 9 \cdot 5^2 = 75 \text{ sq. in.} \]

**Graphs of Rational Functions**

2. To find the vertical asymptotes, set the denominator equal to zero:

\[ y = \frac{5x - 1}{2x - 5} \]

\[ 2x - 6 = 0 \Rightarrow x = 3 \]

\( x = 3 \) is the vertical asymptote.

To find the horizontal asymptote, keep only highest powers of \( x \):

\[ y = \frac{5x - 1}{2x - 5} \]

\[ y = \frac{5x}{2x} = \frac{5}{2} \]

\( y = 5/2 \) is the horizontal asymptote.

4. To find the vertical asymptotes, set the denominator equal to zero:

\[ y = \frac{4x^2}{4x^2 + 1} \]

However, \( 4x^2 + 1 = 0 \) has no real solution, so there are no vertical asymptotes.
To find the horizontal asymptote, keep only highest powers of $x$:

$$y = \frac{4x^2}{4x^2 + 1}$$
$$y = \frac{4x^2}{4x^2} = 1$$

$y = 1$ is the horizontal asymptote.

6. To find the vertical asymptotes, set the denominator equal to zero:

$$y = \frac{3x^2}{x^2 - 4}$$
$$x^2 - 4 = 0 \Rightarrow (x + 2)(x - 2) = 0$$

$x = \pm 2$ are the vertical asymptotes.

To find the horizontal asymptote, keep only highest powers of $x$:

$$y = \frac{3x^2}{x^2 - 4}$$
$$y = \frac{3x^2}{x^2} = 3$$

$y = 3$ is the horizontal asymptote.

8. To find the vertical asymptotes, set the denominator equal to zero:

$$y = \frac{2x + 5}{x^2 - 2x - 8}$$
$$x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0$$

$x = 4$ and $x = -2$ are the vertical asymptotes.

To find the horizontal asymptote, notice that the highest power of $x$ is in the denominator:

$$y = \frac{2x + 5}{x^2 - 2x - 8}$$

$y = 0$ is the horizontal asymptote.
14. To find the vertical asymptotes, set the denominator equal to zero:

\[ y = \frac{x}{x^2 + 9} \]

But the equation \( x^2 + 9 = 0 \) has no real solutions. Therefore there are no vertical asymptotes.

To find the horizontal asymptote, notice that the highest power of \( x \) is in the denominator:

\[ y = \frac{x}{x^2 + 9} \]

\( y = 0 \) is the horizontal asymptote.

16.
22. We use the formula that relates voltage, current and resistance:

\[ L_{total} = \frac{V_{total}}{R_{total}} \]

Plug in the known values: \( I = 1.2, \ V = 27 \).

\[ 1.2 = \frac{27}{R_{total}} \]

Multiply both sides by \( R \):

\[ 1.2 \ R_{total} = 27 \]

Divide both sides by 1.2:
\[ R_{\text{total}} = \frac{27}{1.2} = 22.5 \]

Since \( R_{\text{total}} = X + 10 \), we have

\[ R_{\text{total}} = X + 10 = 22.5 \]

\[ X = 12.5 \text{ ohms} \]

24. Note that the way the resistances are set-up, we have \( R_{\text{total}} = 5 + r_{\text{total}} \), where the resistance \( r \) comes from those in parallel. First we use the formula that relates voltage, current and resistance to find \( R \):

\[ I_{\text{total}} = \frac{V_{\text{total}}}{R_{\text{total}}} \]

Plug in the known values: \( I = 2.4 \), \( V = 24 \).

\[ 2.4 = \frac{24}{R_{\text{total}}} \]

Multiply both sides by \( R \):

\[ 2.4 \cdot R_{\text{total}} = 24 \]

Divide both sides by 2.4:

\[ R_{\text{total}} = \frac{24}{2.4} = 10 \]

Now find the resistance in parallel:

\[ \frac{1}{r_{\text{total}}} = \frac{1}{X} + \frac{1}{10} \]

Solve for \( r \):

\[ 10X \left( \frac{1}{r_{\text{total}}} \right) = 10X \left( \frac{1}{X} + \frac{1}{10} \right) \]

\[ \frac{10X}{r_{\text{total}}} = 10 + X \]

\[ \frac{10X}{10 + X} = r_{\text{total}} \]

Putting it all together:

\[ R_{\text{total}} = 5 + r_{\text{total}} \]

\[ 10 = 5 + \frac{10X}{10 + X} \]

\[ 5 = \frac{10X}{10 + X} \]

\[ 5(10 + X) = 10X \]

\[ 50 + 5X = 10X \]

\[ 50 = 5X \]

\[ X = 10 \text{ ohms} \]
Division of Polynomials

NO REVIEW QUESTIONS

Rational Expressions

2. \( \frac{x^2 + 2x}{x} = \frac{x(x+2)}{x} = x + 2, \ x \neq 0 \)

4. \( \frac{6x^2 + 2x}{4x} = \frac{2x(3x+1)}{4x} = \frac{3x+1}{2}, \ x \neq 0 \)

6. \( \frac{x^2 - 9}{6x + 15} = \frac{(x+3)(x-3)}{3(x+3)} = \frac{x-3}{3}, \ x \neq -3 \)

8. \( \frac{2x^2 + 10x}{x^2 + 10x + 25} = \frac{2x(x+5)}{(x+5)^2} = \frac{2x}{x+5} \)

10. \( \frac{x^2 - 16}{x^2 + 2x - 8} = \frac{(x+4)(x-4)}{(x+4)(x-2)} = \frac{x-4}{x-2}, \ x \neq -4 \)

12. \( \frac{x^3 + x^2 - 20x}{6x^2 + 6x - 120} = \frac{x(x^2 + x - 20)}{6(x^2 + x - 20)} = \frac{x}{6}, \ x \neq -5, \ x \neq 4 \)

14. When we set the denominator to zero we obtain the excluded value:

\[
x + 2 = 0
\]
\[
\Rightarrow x = -2
\]

The excluded value is \(-2\).

16. Factor the denominator:

\[
\frac{3x + 1}{x^2 - 4} = \frac{3x + 1}{(x+2)(x-2)}
\]

When we set the denominator to zero we obtain the excluded value:

\[
(x + 2)(x - 2) = 0
\]
\[
\Rightarrow x + 2 = 0
\]
\[
\Rightarrow x - 2 = 0
\]

The excluded values are 2 and \(-2\).

18. Factor the denominator:

\[
\frac{2x^2 + 3x - 1}{x^2 - 3x - 28} = \frac{2x^2 + 3x - 1}{(x+4)(x-7)}
\]

When we set the denominator to zero we obtain the excluded value:

\[
(x + 4)(x - 7) = 0
\]
\[
\Rightarrow x + 4 = 0
\]
\[
\Rightarrow x - 7 = 0
\]

The excluded values are \(-4\) and 7.

20. Factor the denominator:
\[ \frac{9}{x^3 + 11x^2 + 30x} = \frac{9}{x(x^2 + 11x + 30)} = \frac{9}{x(x + 6)(x + 5)} \]

When we set the denominator equal to zero we obtain the excluded value:

\[ x(x + 6)(x + 5) = 0 \]
\[ \Rightarrow x = 0 \]
\[ \Rightarrow (x + 6) = 0 \]
\[ \Rightarrow (x + 5) = 0 \]

The excluded values are 0, −5, −6.

22. Since the denominator does not factor over the integers, we set the denominator equal to zero and solve using the quadratic formula:

\[ 3x^2 - 2x - 4 = 0 \]
\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)} = \frac{2 \pm \sqrt{4 + 48}}{6} = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm \sqrt{13 \cdot 4}}{6} = \frac{2 \pm 2 \sqrt{13}}{6} = \frac{1 \pm \sqrt{13}}{3} \]

The excluded values are \( \frac{1 \pm \sqrt{13}}{3} \).

30. The ratio of the two volumes is

\[ \frac{\frac{4}{3} \pi R^3}{\frac{4}{3} \pi (R - 4)^3} \]

Simplify:

\[ = \frac{R^3}{(R - 4)^3}. \]

24. Since the denominator does not factor over the integers:

\[ \frac{12}{x^2 + 6x + 1} \]

Set the denominator equal to zero and solve using the quadratic formula. We obtain the excluded values:

\[ x^2 + 6x + 1 = 0 \]
\[ x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 4}}{2} = \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4 \sqrt{2}}{2} = -3 \pm 2 \sqrt{2} \]

The excluded values are \( -3 \pm 2 \sqrt{2} \).

26. Start with the Law of Gravitation formula:

\[ F = G \frac{m_1 m_2}{d^2} \]

Plug in the known values:
\[ 20 = G \frac{m_1 m_2}{d^2} \]

Since the distance now gets doubled, we have:

\[ F = G \frac{m_1 m_2}{(2d)^2} \]

Simplify:

\[ F = G \frac{m_1 m_2}{4d^2} = \frac{1}{4} \cdot G \frac{m_1 m_2}{d^2} = \frac{1}{4} \cdot 20 = 5 \]

So the new force of attraction between the objects is 5 N.

27. Start with the Law of Gravitation formula:

\[ F = G \frac{m_1 m_2}{d^2} \]

Plug in the known values:

\[ 36 = G \frac{m_1 m_2}{d^2} \]

Since everything now gets doubled, we have:

\[ F = G \frac{(2m_1)(2m_2)}{(2d)^2} \]

Simplify:

\[ F = G \frac{4m_1 m_2}{4d^2} = G \frac{m_1 m_2}{d^2} = 36 \]

So the new force of attraction between the objects is still 36 N.

28. The ratio of the surface area of a sphere to its volume is:

\[ \frac{4\pi R^2}{\frac{4}{3} \pi R^3} = \]

Simplify:

\[ \frac{4R^2}{\frac{4}{3} R^3} = \frac{4}{\frac{4}{3}} \frac{R}{R} = \frac{3}{1} \frac{R}{R} = 3 \]
Multiplication and Division of Rational Expressions

2. \(\frac{2 \cdot 2x^2}{y} = 2xy \cdot \frac{y}{2x^2} = \frac{y^2}{x}\)

4. \(\frac{2xy}{x^2} = \frac{4y}{x^3} = \frac{4y^3}{x^2}\) (note SE has typo here, writing \(47^3\) instead of \(4y^3\))

6. \(\frac{6ab}{a^2} \cdot \frac{a^3b^2}{3b^2} = \frac{6a^4b^2}{3a^2b^2} = 2a^2\).

8. \(\frac{33}{5} \cdot \frac{20}{11a^2} = \frac{660a^2}{55a^2} = 12\)

10. \(\frac{2x^2 + 2x - 24}{x^2 + 3x} \cdot \frac{x^2 + x - 6}{x + 4} = \frac{2(x^2 + x - 12)}{x(x + 3)} \cdot \frac{x^2 + x - 6}{x + 4} = \frac{2(x + 4)(x - 3)}{x(x + 3)} \cdot \frac{(x + 3)(x - 2)}{x + 4} = \frac{2(x - 3)}{x} \cdot \frac{(x - 2)}{1} = \frac{2(x - 3)(x - 2)}{x}\)

12. \(\frac{x^2 - 25}{x + 3} \div (x - 5) = \frac{x^2 - 25}{x + 3} \cdot \frac{1}{(x - 5)} = \frac{(x - 5)(x + 5)}{x + 3} \cdot \frac{1}{x - 5} = \frac{x^2 - 25}{x + 3} \div (x - 5) = \frac{x^2 - 25}{x + 3} \cdot \frac{1}{(x - 5)} = \frac{(x + 5)}{x + 3} \cdot \frac{1}{1} = \frac{x + 5}{x + 3}\)

14. \(\frac{x}{x - 5} \cdot \frac{x^2 - 8x + 15}{x^2 - 3x} = \frac{x}{x - 5} \cdot \frac{(x - 3)(x - 5)}{x(x - 3)} = \frac{x}{x - 5} \cdot \frac{(x - 5)}{x} = 1\)

16. \(\frac{5x^2 + 16x + 3}{36x^2 - 25} \cdot (6x^2 + 5x) = \frac{5x^2 + 16x + 3}{36x^2 - 25} \cdot \frac{6x^2 + 5x}{1} = \frac{5x^2 + 16x + 3}{36x^2 - 25} \cdot \frac{6x^2 + 5x}{1} = \frac{(5x + 1)(x + 3)}{(6x - 5)(6x + 5)} \cdot \frac{x(6x + 5)}{1} = \frac{(5x + 1)(x + 3)}{(6x - 5)(6x + 5)} \cdot \frac{x}{1} = \frac{x(5x + 1)(x + 3)}{6x - 5}\)
18. 
\[
\frac{x^2 + x - 12}{x^2 + 4x + 4} \div \frac{x - 3}{x + 2} = \frac{x^2 + x - 12}{x^2 + 4x + 4} \cdot \frac{x + 2}{x - 3}
\]
\[
= \frac{(x + 4)(x - 3)}{(x + 2)(x + 2)} \cdot \frac{x + 2}{x - 3}
\]
\[
= \frac{x + 4}{x + 2}
\]

20. 
\[
\frac{x^2 + 8x + 16}{7x^2 + 9x + 2} \div \frac{7x + 2}{x^2 + 4x} = \frac{x^2 + 8x + 16}{7x^2 + 9x + 2} \cdot \frac{x^2 + 4x}{7x + 2}
\]
\[
= \frac{(x + 4)^2}{(7x + 2)(x + 1)} \cdot \frac{x(x + 4)}{7x + 2}
\]
\[
= \frac{x(x + 4)^3}{(7x + 2)^2(x + 1)}
\]

22. Let distance of the trip = \(d\), initial speed = \(s\), and initial time = \(t_1\).
Then, \(d = st_1\)

or

\[
\frac{d}{s} = t_1
\]

On the return trip, we have: distance of the trip = \(d\), initial speed = \(s + 15\), and initial time = \(t_2\).

Then, \(d = (s + 15) t_2\)

or

\[
\frac{d}{s + 15} = t_2
\]

To find the factor the driving time decreased, simplify the ratio:

\[
\frac{t_2}{t_1} = \frac{d(s + 15)}{d/s} = \frac{d}{s} \cdot \frac{s}{d} = \frac{s}{s + 15}
\]

Addition and Subtraction of Rational Expressions

2. \(\frac{10}{21} + \frac{9}{35} = \frac{10 \cdot 5}{21 \cdot 5} + \frac{9 \cdot 3}{35 \cdot 3} = \frac{50 + 21}{105} = \frac{71}{105}\)

4. \(\frac{3x - 1}{x + 9} - \frac{4x + 2}{x + 9} = \frac{3x - 1 - (4x + 2)}{x + 9} = \frac{-x - 3}{x + 9}\)

6. \(\frac{x^2}{x + 5} - \frac{25}{x + 5} = \frac{x^2 - 25}{x + 5} = \frac{(x + 5)(x - 5)}{x + 5} = x - 5\)

8. \(\frac{10}{3x - 1} - \frac{7}{1 - 3x} = \frac{10}{3x - 1} - \frac{7}{(-1)(3x - 1)} = \frac{10}{3x - 1} = \frac{7}{-(3x - 1)} = \frac{10}{3x - 1} + \frac{7}{3x - 1} = \frac{17}{3x - 1}\)

10.
\[
\frac{5x + 1}{x + 4} + 2 = \frac{5x + 1}{x + 4} + 2 \cdot \frac{x + 4}{x + 4}
\]
\[
= \frac{5x + 1 + 2(x + 4)}{x + 4}
\]
\[
= \frac{5x + 1 + 2x + 8}{x + 4}
\]
\[
= \frac{7x + 9}{x + 4}
\]

12. \( \frac{4}{5x^2} - \frac{2}{7x^3} = \frac{7x}{5x^2} \cdot \frac{4}{7x^3} - \frac{5}{x} \cdot \frac{2}{7x^3} = \frac{28x - 10}{35x^3} \)

14. \[
\frac{10}{x + 5} + \frac{2}{x + 2} = \frac{x + 2}{x + 5} \cdot \frac{10}{x + 2} + \frac{x + 5}{x + 2} \cdot \frac{2}{x + 5}
\]
\[
= \frac{(10)(x + 2)}{(x + 2)(x + 5)} + \frac{(2)(x + 5)}{(x + 2)(x + 5)}
\]
\[
= \frac{(10)(x + 2) + (2)(x + 5)}{(x + 2)(x + 5)}
\]
\[
= \frac{10x + 20 + 2x + 10}{(x + 2)(x + 5)}
\]
\[
= \frac{12x + 30}{(x + 2)(x + 5)}
\]

16. \[
\frac{4x - 3}{2x + 1} + \frac{x + 2}{x - 9} = \frac{x - 9}{2x + 1} \cdot \frac{4x - 3}{x - 9} + \frac{2x + 1}{x - 9} \cdot \frac{x + 2}{x - 9}
\]
\[
= \frac{(4x - 3)(x - 9)}{(2x + 1)(x - 9)} + \frac{(2x + 1)(x + 2)}{(x - 9)}
\]
\[
= \frac{(4x^2 - 36x - 3x + 27) + (2x^2 + x + 4x + 2)}{(2x + 1)(x - 9)}
\]
\[
= \frac{6x^2 - 34 + 29}{(2x + 1)(x - 9)}
\]

18. \[
\frac{2}{5x + 2} + \frac{x + 1}{x^2} = \frac{x^2}{x^2} \cdot \frac{2}{5x + 2} - \frac{5x + 2}{x^2} \cdot \frac{x + 1}{x^2}
\]
\[
= \frac{(x^2)(2)}{(x^2)(5x + 2)} - \frac{(5x + 2)(x + 1)}{(x^2)(5x + 2)}
\]
\[
= \frac{(x^2)(2) - (5x + 2)(x + 1)}{x^2(5x + 2)}
\]
\[
= \frac{2x^2 - (5x^2 + 7x + 2)}{x^2(5x + 2)}
\]
\[
= \frac{-3x^2 + 7x + 2}{x^2(5x + 2)}
\]

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20. \[ \frac{5x + 3}{x^2 + x} + \frac{2x + 1}{x} = \frac{5x + 3}{x(x + 1)} + \frac{2x + 1}{x} \]
\[ = \frac{5x + 3}{x(x + 1)} + \frac{x + 1}{x + 1} \cdot \frac{2x + 1}{x} \]
\[ = \frac{5x + 3}{x(x + 1)} + \frac{(x + 1)(2x + 1)}{(x + 1)x} \]
\[ = \frac{5x + 3}{x(x + 1)} + \frac{(x + 1)(2x + 1)}{x(x + 1)} \]
\[ = \frac{(5x + 3) + (2x^2 + 2x + 1)}{x(x + 1)} \]
\[ = \frac{2x^2 + 8x + 4}{x(x + 1)} \]
\[ = \frac{2(x^2 + 4x + 2)}{x(x + 1)} \]

22. \[ \frac{2x}{(x + 2)(3x - 4)} + \frac{7x}{(3x - 4)^2} = \frac{3x - 4}{(x + 2)(3x - 4)} \cdot \frac{2x}{(3x - 4)^2} + \frac{x + 2}{(3x - 4)^2} \]
\[ = \frac{(2x)(3x - 4)}{x + 2)(3x - 4)^2} + \frac{(7x)(x + 2)}{(3x - 4)^2} \]
\[ = \frac{(2x)(3x - 4) + (7x)(x + 2)}{(x + 2)(3x - 4)^2} \]
\[ = \frac{6x^2 - 8x + 7x^2 + 14x}{(x + 2)(3x - 4)^2} \]
\[ = \frac{13x^2 + 6x}{(x + 2)(3x - 4)^2} \]

24. \[ \frac{1}{(x - 2)(x - 3)} + \frac{4}{(2x + 5)(x - 6)} = \frac{(2x + 5)(x - 6)}{(2x + 5)(x - 6) \cdot (x - 2)(x - 3)} + \frac{1}{(x - 2)(x - 3)} \cdot \frac{4}{(2x + 5)(x - 6)} \]
\[ = \frac{2x^2 + 5x - 12x - 30}{(2x + 5)(x - 6)(x - 2)(x - 3)} + \frac{x^2 - 2x - 3x + 6}{(x - 2)(x - 3)(2x + 5)(x - 6)} \]
\[ = \frac{2x^2 + 5x - 12x - 30 + x^2 - 2x - 3x + 6}{(2x + 5)(x - 6)(x - 2)(x - 3)} \]
\[ = \frac{3x^2 - 12x - 24}{(2x + 5)(x - 6)(x - 2)(x - 3)} \]
\[ = \frac{3(x^2 - 4x - 8)}{(2x + 5)(x - 6)(x - 2)(x - 3)} \]

26. \[ -\frac{x^2}{x^2 - 7x + 6} + (x - 4) = \frac{-x^2}{(x - 6)(x - 1)} + \frac{(x - 6)(x - 1)}{(x - 6)(x - 1)} \cdot (x - 4) \]
\[ = \frac{-x^2}{(x - 6)(x - 1)} + \frac{(x - 6)(x - 1)(x - 4)}{(x - 6)(x - 1)} \]
\[ = -x^2 + \frac{(x - 6)(x - 1)(x - 4)}{(x - 6)(x - 1)} \]
\[ = -x^2 + \frac{(x^3 - 11x^2 + 34x - 24)}{(x - 6)(x - 1)} \]
\[ = \frac{x^3 - 12x^2 + 34x - 24}{(x - 6)(x - 1)} \]

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(note SE is clearly incorrect here, as the numerator must have at least a cubic term)

28. \[
\frac{1}{x^2 - 9} + \frac{2}{x^2 + 5x + 6} = \frac{1}{(x + 3)(x - 3)} + \frac{2}{(x + 3)(x + 2)}
\]
\[
= \frac{x + 2}{x + 2} \cdot \frac{1}{(x + 3)(x - 3)} + \frac{x - 3}{x - 3} \cdot \frac{2}{(x + 3)(x + 2)}
\]
\[
= \frac{1}{(x + 2)(x + 3)(x - 3)} + \frac{2}{(x + 2)(x + 3)(x - 3)}
\]
\[
= \frac{3x - 4}{(x + 2)(x + 3)(x - 3)}
\]

30. \[
\frac{4}{9x^2 - 49} - \frac{1}{3x^2 + 5x - 28} = \frac{4}{(3x + 7)(3x - 7)} - \frac{1}{(3x - 7)(x + 4)}
\]
\[
= \frac{x + 4}{x + 4} \cdot \frac{4}{(3x + 7)(3x - 7)} - \frac{(3x + 7)}{(3x + 7)} \cdot \frac{1}{(3x - 7)(x + 4)}
\]
\[
= \frac{4x + 16}{(x + 4)(3x + 7)(3x - 7)} - \frac{3x + 7}{(3x + 7)(3x - 7)(x + 4)}
\]
\[
= \frac{(x + 4)(3x + 7)(3x - 7)}{(x + 4)(3x + 7)(3x - 7)}
\]
\[
= \frac{3x - 4}{(x + 2)(x + 3)(x - 3)}
\]

32. Let \(x\) and \(8x\) be the two numbers. \((Note: SE should read “8 times another,” not “8 times more than another”)\) We have the equation

\[
\frac{1}{x} - \frac{1}{8x} = \frac{21}{20}
\]

Solve this by multiplying both sides by \(40x\):

\[
\frac{1}{x}(40x) - \frac{1}{8x}(40x) = \frac{21}{20}(40x)
\]
\[
40 - 5 = 42x
\]
\[
35 = 42x
\]
\[
\frac{5}{6} = x
\]

So the numbers are \(5/6\) and \(20/3\) \((Note: the negatives of these numbers is another possible solution since SE doesn't specify which order they are subtracted in.)\)

34. In \(2.5\) hours, Stephan has washed \(\frac{25}{6} = \frac{5}{12}\) of the cars. There remains a fraction of \(\frac{7}{12}\) of the cars to be washed. Using the fact that:

\[
\text{Part of the task to be completed} = \text{rate of work} \cdot \text{time spent on the task}
\]

Misha can complete the job in:
\[
\frac{7}{12} = \frac{1}{5}t
\]

Solve for \( t \):

\[
\frac{35}{12} = t
\]

Answer: Misha completes the job in 2 hours and 55 minutes.

Check: The answer is reasonable.

36. After four hours, the machines have completed a combined

\[
\left(\frac{1}{10} + \frac{1}{14}\right)(4) = \frac{4}{10} + \frac{4}{14} = \frac{2}{5} + \frac{2}{7} = \frac{14}{35} + \frac{10}{35} = \frac{24}{35}
\]

of the daily quota.

The remaining \( 1 - \frac{24}{35} = \frac{11}{35} \) of the daily quota is completed by the faster machine. It works in:

\[
\frac{11}{35} = \frac{1}{10}t
\]
\[
\frac{110}{35} = t
\]
\[
\frac{22}{7} = t \text{ hours}
\]

Answer: The faster machine filled the daily quota in approximately 3 hours and 9 minutes.

Check: The answer is reasonable.

**Solutions of Rational Equations**

2.

\[
\frac{4x}{x + 2} = \frac{5}{9}
\]

\[(4x)(9) = (5)(x + 2)\]

\[36x = 5x + 10\]

\[31x = 10\]

\[x = \frac{10}{31}\]

4.

\[
\frac{7x}{x - 5} = \frac{x + 3}{x}
\]

\[7x^2 = (x + 3)(x - 5)\]

\[7x^2 = x^2 + 3x - 5x - 15\]

\[7x^2 = x^2 - 2x - 15\]

\[6x^2 + 2x + 15 = 0\]

The discriminant is negative:

\[b^2 - 4ac = 2^2 - 4(6)(15) < 0\]

implying that the quadratic and hence the original equations have no real solutions.
6. 
\[
\frac{3x^2 + 2x - 1}{x^2 - 1} = 2
\]
\[
3x^2 + 2x - 1 = (-2)(x^2 - 1)
\]
\[
3x^2 + 2x - 1 = -2x^2 + 2
\]
\[
5x^2 + 2x - 3 = 0
\]
\[
(5x - 3)(x + 1) = 0
\]
\[
5x + 3 = 0 \Rightarrow x = \frac{3}{5}
\]
\[
x - 1 = 0 \Rightarrow x = -1
\]
However, \(x = -1\) is excluded because the denominator vanishes there. So the only solution is \(x = \frac{3}{5}\).

8. 
\[
-3 \cdot \frac{1}{x+1} = \frac{2}{x}
\]
\[
x(x+1)\left(-3 \cdot \frac{1}{x+1}\right) = x(x+1)\left(\frac{2}{x}\right)
\]
\[
-3x(x+1) + x = 2(x+1)
\]
\[
-3x^2 - 3x + x = 2x + 2
\]
\[
-3x^2 - 4x - 2 = 0
\]
\[
3x^2 + 4x + 2 = 0
\]
The discriminant is negative:
\[
b^2 - 4ac = 4^2 - 4(3)(2) < 0
\]
Which implies that the quadratic and hence the original equations have no solutions.

10. 
\[
\frac{3}{2x-1} + \frac{2}{x+4} = 2
\]
\[
(2x-1)(x+4)\left(\frac{3}{2x-1} + \frac{2}{x+4}\right) = (2x-1)(x+4)(2)
\]
\[
3(x+4) + 2(2x-1) = (2x-1)(x+4)(2)
\]
\[
3x + 12 + 4x - 2 = 4x^2 + 14x - 8
\]
\[
4x^2 + 7x - 18 = 0
\]
This equation does not factor over the integers, so use the quadratic formula:
\[
x = \frac{-7 \pm \sqrt{7^2 - 4(4)(-18)}}{2(4)} = \frac{-7 \pm \sqrt{49 + 288}}{8} = \frac{-7 \pm \sqrt{337}}{8}
\]
\[
x \approx -3.17, 1.42
\]

12. 
\[
\frac{x+1}{x-1} + \frac{x-4}{x+4} = 3
\]
\[
(x-1)(x+4)\left(\frac{x+1}{x-1} + \frac{x-4}{x+4}\right) = (x-1)(x+4)(3)
\]
\[
(x+1)(x+4) + (x-4)(x-1) = (x-1)(x+4)(3)
\]
\[
x^2 + 5x + 4 + x^2 - 5x + 4 = 3x^2 + 9x - 12
\]
\[
x^2 + 9x - 20 = 0
\]
This equation does not factor over the integers, so use the quadratic formula:

\[
x = \frac{-9 \pm \sqrt{9^2 - 4(1)(-20)}}{2(1)} = \frac{-9 \pm \sqrt{81 + 80}}{2} = \frac{-9 \pm \sqrt{161}}{2}
\]

\[x \approx -10.84, 1.84\]

14.

\[
\frac{2}{x^2 + 4x + 3} = 2 + \frac{x - 2}{x + 3}
\]

\[
\frac{2}{(x + 3)(x + 1)} = \frac{2 + x - 2}{x + 3}
\]

\[
(x + 3)(x + 1) \left( \frac{2}{(x + 3)(x + 1)} \right) = (x + 3)(x + 1) \left( 2 + \frac{x - 2}{x + 3} \right)
\]

\[= 2 = 2(x + 3)(x + 1) + (x - 2)(x + 1)\]

\[= 2 = 2x^2 + 8x + 6 + x^2 - x - 2\]

\[3x^2 + 7x + 2 = 0\]

\[(3x + 1)(x + 2) = 0\]

\[(3x + 1) = 0 \Rightarrow x = -\frac{1}{3}\]

\[(x + 2) = 0 \Rightarrow x = -2\]

Both solutions check out.

16.

\[
\frac{x}{x^2 - 36} + \frac{1}{x - 6} = \frac{1}{x + 6}
\]

\[
\frac{x}{(x + 6)(x - 6)} + \frac{1}{x - 6} = \frac{1}{x + 6}
\]

\[
(x + 6)(x - 6) \left( \frac{x}{(x + 6)(x - 6)} + \frac{1}{x - 6} \right) = (x + 6)(x - 6) \left( \frac{1}{x + 6} \right)
\]

\[x + (x + 6) = (x - 6)\]

\[2x + 6 = x - 6\]

\[x = -12\]

The solution checks out.

18.

\[
\frac{-x}{x - 2} + \frac{3x - 1}{x + 4} = \frac{1}{x^2 + 2x - 8}
\]

\[
\frac{-x}{x - 2} + \frac{3x - 1}{x + 4} = \frac{1}{(x + 4)(x - 2)}
\]

\[
(x + 4)(x - 2) \left( \frac{-x}{x - 2} + \frac{3x - 1}{x + 4} \right) = (x + 4)(x - 2) \left( \frac{1}{(x + 4)(x - 2)} \right)
\]

\[-x(x + 4) + (3x - 1)(x - 2) = 1\]

\[-x^2 + 4x + 3x^2 - 7x + 2 = 1\]

\[2x^2 - 11x + 1 = 0\]

This equation does not factor over the integers, so use the quadratic formula:

\[
x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(1)}}{2(2)} = \frac{11 \pm \sqrt{121 - 8}}{4} = \frac{11 \pm \sqrt{113}}{4}
\]

\[x \approx 5.41, 0.092\]
20. **Define variables:**
Let \( s = \) speed of the current

**Construct a table:** We make a table that displays the information we have in a clear manner:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Distance (miles)</th>
<th>Rate (mph)</th>
<th>Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>downstream</td>
<td>60</td>
<td>( 20 + s )</td>
<td>( t )</td>
</tr>
<tr>
<td>upstream</td>
<td>40</td>
<td>( 20 - s )</td>
<td>( t )</td>
</tr>
</tbody>
</table>

**Write an equation:** We know that

The time to go downstream is:

\[
\frac{60}{20 + s} = t
\]

The time to go upstream is:

\[
\frac{40}{20 - s} = t
\]

We also know that the total times are the same:

\[
\frac{60}{20 + s} = \frac{40}{20 - s}
\]

Solve the equation:

\[
60(20 - s) = 40(20 + s)
\]

\[
1200 - 60s = 800 + 40s
\]

\[
400 = 100s
\]

\[
s = 4 \text{ mph}
\]

Check: The answer checks out.

22. **Define variables:**
Let \( s = \) speed of the airplane when there is no wind.

**Construct a table:** We make a table that displays the information we have in a clear manner:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Distance (miles)</th>
<th>Rate (mph)</th>
<th>Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>against the wind</td>
<td>300</td>
<td>( s - 30 )</td>
<td>( t )</td>
</tr>
<tr>
<td>with the wind</td>
<td>420</td>
<td>( s + 30 )</td>
<td>( t )</td>
</tr>
</tbody>
</table>
Write an equation: We know that

The time to go against the wind is:

\[ 300 = (s - 30)t \]
\[ \frac{300}{s - 30} = t \]

The time to go with the wind is:

\[ 420 = (s + 30)t \]
\[ \frac{420}{s + 30} = t \]

We also know that the total times are the same:

\[ \frac{300}{s - 30} = \frac{420}{s + 30} \]

Solve the equation:

\[ 300(s + 30) = 420(s - 30) \]
\[ 300s + 9000 = 420s - 12600 \]
\[ 120s = 21600 \]
\[ s = 180 \text{ mph} \]

Check: The answer checks out.

24. Define variables:

Let \( n \) = the number of members belonging to the non-profit organization.

Construct a table: We make a table that displays the information we have in a clear manner:

<table>
<thead>
<tr>
<th>Groups</th>
<th>Members</th>
<th>Donations (per member)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original group</td>
<td>( n )</td>
<td>2250 ( \frac{2250}{n} )</td>
</tr>
<tr>
<td>New group</td>
<td>( n + 30 )</td>
<td>250 ( \frac{250}{n + 30} )</td>
</tr>
</tbody>
</table>

Write an equation: We know that

If there were thirty more members, then each member could contribute $20 less:

\[ \frac{2250}{n + 30} = \frac{2250}{n} - 20 \]

Solve the equation:
\[ n(n + 30) \left( \frac{2250}{n + 30} \right) = n(n + 30) \left( \frac{2250}{n} - 20 \right) \]
\[ 2250n = 2250(n + 30) - 20n(n + 30) \]
\[ 2250n = 2250n + 67500 - 20n^2 - 600n \]
\[ 20n^2 + 600n - 67500 = 0 \]
\[ n^2 + 30n - 3375 = 0 \]

Solve by quadratic formula:

\[ n = \frac{-30 \pm \sqrt{30^2 - (1)(-3375)}}{2(1)} \]
\[ n = \frac{-30 \pm \sqrt{14400}}{2} \]
\[ n = \frac{-30 \pm 120}{2} \]
\[ \Rightarrow n = \frac{-30 + 120}{2} = 45 \]
\[ \Rightarrow n = \frac{-30 - 120}{2} = -75 \]

Answer: There are 45 members.
Check: The answer checks out.

**Surveys and Samples**

2a. The sample is biased because only people who can afford a five star restaurant are interviewed. The sampling could be improved by constructing a stratified sample with categories based on income level and the interviews taking place in various locations.

2b. This will give a one-sided view of the issue. The study can be improved by also interviewing drivers passing through the intersection.

2c. This should be an unbiased sample and give valid results. It could be improved by creating a stratified samples with categories based on race or cultural background.

2d. Although the sample is stratified, the sample is biased because the survey would only find the opinion of the students. A better survey would also include the opinion of teachers and parents.

2e. This would be a biased sample because it would only show the opinion of the people using public transportation. The study could be improved by creating a sample that includes people that drive on the bus routes.

2f. Although the sample is stratified it is still biased. A better sample would include the opinion of parents and teachers also.

4.
6. a.
8. Answers may vary.

10. Answers may vary.